KD EDUCATION ACADEMY [9582701165]

Time : 6 Hour | Physics | Total Marks : 370

kd 90+ questions ch-3 motion in a plane

*** Choose The Right Answer From The Given Options.[1 Marks Each] [35]**

1. The horizontal range of a projectile fired at an angle of 15° is 50m. If it is fired with the same speed at an angle of 45°, its range will be

(A) 60m (B) 71m (C) 100m (D) 141m

Ans. :

c. 100m

Explanation:

projectile is fired at $\theta = 15^\circ,\ \mathrm{R} = 50\mathrm{m}$

$\mathrm{R} = \frac{\mathrm{u}^2 \sin 2\theta}{\mathrm{g}}$

$50 = \frac{\mathrm{u}^2 \sin 2\times 15^\circ}{\mathrm{g}} \Rightarrow \mathrm{u}^2 = 50\mathrm{g} \times 2$

$\mathrm{u}^2 = 100\mathrm{g}$

Now $\theta = 45^\circ,\ \mathrm{u}^2 = 100\mathrm{g}$

$\therefore\ \mathrm{R} = \frac{\mathrm{u}^2 \sin 2\theta}{\mathrm{g}} = \frac{100\mathrm{g} \times \sin 2\times 45^\circ}{\mathrm{g}}$

$\Rightarrow\ \mathrm{R} = 100\mathrm{m}$

So, this verifies option (c).

2. From the top of a tower of height 40m, a ball is projected upwards with a speed of 20m/ s at an angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight time taken to come back to the same elevation) is (Take g = 10m/ s^2)

(A) 2 : 1 (B) 3 : 1 (C) 3 : 2 (D) 1.5 : 1

Ans. :

a. 2 : 1

3. A vector is of magnitude $10\sqrt{3}$ units and making equal angles with the positive direction of x, y and z axis is:

(A) $10(\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}})$

(B) $10(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$

(C) $10(-\hat{\mathrm{i}} - \hat{\mathrm{j}} - \hat{\mathrm{k}})$

(D) $10(\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}})$

Ans. :

a. $10(\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}})$

4. The speed of a projectile at the maximum height is $\frac{1}{2}$ its initial speed. Find the ratio of range of projectile to the maximum height attained.

(A) $4\sqrt{3}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{4}$ (D) 6

Ans. :

b. $\frac{4}{\sqrt{3}}$

5. The sum of magnitudes of two forces acting at a point is 18 units and the magnitude oftheir resultant is 12 units. The resultant is at 90° with the force of the smaller magnitude. The magnitude of the individual forces is:

(A) 5, 12 (B) 5, 13

(C) 6, 14 (D) None of these.

Ans. :

b. 5, 13

6. The ceiling of a hall is 30m high. A ball is thrown with 60ms^{-1} at an angle θ, so that maximum horizontal distance may be covered. The angle of projection is given by,

(A) $\sin\theta = \frac{1}{\sqrt{8}}$

(B) $\sin\theta = \frac{1}{\sqrt{6}}$

(C) $\sin\theta = \frac{1}{\sqrt{3}}$

(D) None of these.

Ans. :

b. $\sin\theta = \frac{1}{\sqrt{6}}$

Explanation:

Given u = 60ms^{-1}

$\therefore$ maximum height, $\mathrm{H} = \frac{\mathrm{u}^2 \sin^2\theta}{2\mathrm{g}}$

$\Rightarrow\ 30 = \frac{(60)^2 \sin^2\theta}{2\mathrm{g}}$

$\Rightarrow\ \sin^2\theta = \frac{30\times 2\mathrm{g}}{60\times 60} = \frac{10}{60}$

$\Rightarrow\ \sin\theta = \frac{1}{\sqrt{6}}$

7. The displacement of a particle moving on a circular path of radius r when it makes 60° at the centre is:

(A) 2r (B) r (C) $\sqrt{2}\mathrm{r}$ (D) None of these.

Ans. :

b. r

8. A girl riding a bicycle with a speed of 5ms^{-1} towards North direction sees raindrops falling vertically downwards. On increasing the speed to 15ms rain appears to fall making an angle of 45° of the vertical. Find the magnitude of velocity of rain.

(A) 5ms^{-1} (B) $5\sqrt{5}\ \mathrm{ms}^{-1}$ (C) 25ms^{-1} (D) 10ms^{-1}

Ans. :

b. $5\sqrt{5}\ \mathrm{ms}^{-1}$

9. A body is projected horizontally with a velocity of 4ms^{-1}. The velocity of the body after 0.7s is nearly (take g = 10 ms^{-2})

(A) 10ms^{-1} (B) 8ms^{-1} (C) 19.2ms^{-1} (D) 11ms^{-1}

Ans. :

b. $8ms^{-1}$

10. A boy aims a gun at a target from a point, at a horizontal distance of 100m. If the gun can impart a horizontal velocity of $500ms^{-1}$ to the bullet, the height above the target where he must aim his gun, in order to hit it is (Take g = $10ms^{-2}$)

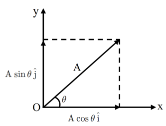
(A) 20cm (B) 10cm (C) 50cm (D) 100cm

Ans. :

a. 20cm

11. . The quantities A_x and A_y are called x and y-components of the vector A. Note that A_x is itself not a vector, but $A_x\hat{i}$ is a vector, and so is $A_y\hat{j}$. Using simple trigonometry, we can express A_x and A_y in terms of the magnitude of A and the angle it makes with the x-axis $A_x = A\cos\theta$ $A_y = A\sin\theta$ Choose the correct figure on the basis of given description.

(A)



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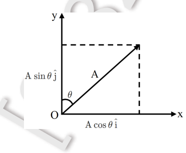
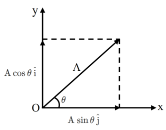


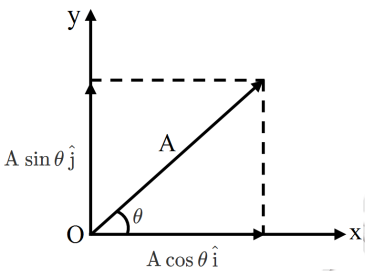
(C)



(D) None of these.

Ans. :

a.



12. Two projectiles A and B thrown with speeds in the ratio $1 : \sqrt{2}$ acquired the same height. If A is thrown at an angle of 45° with the horizontal, then angle of projection of B will be:

(A) 0° (B) 60° (C) 30° (D) 45°

Ans. :

c. 30°

13. Given, |A + B| = P, |A - B| = Q. The value of $P^2 + Q^2$ is:

(A) $2(A^2 + B^2)$ (B) $A^2 - B^2$ (C) $A^2 + B^2$ (D) $2(A^2 - B^2)$

Ans. :

a. $2(A^2 + B^2)$

14. A body is thrown with a velocity of $10ms^{-1}$ at an angle of 60° with the horizontal. Its velocity at the highest point is:

(A) zero
(B) $5ms^{-1}$
(C) $10ms^{-1}$
(D) $8.66ms^{-1}$

Ans. :

b. $5ms^{-1}$

Explanation:

At the highest point of the angular projection, the velocity of projectile has only horizontal component velocity $= \mathrm{u}\cos\theta = 10\cos 60^{\circ} = 5\mathrm{ms}^{-1}$.

15. During projectile motion the quantities that remain unchanged are:

(A) Force and vertical velocity.
(B) Acceleration and horizontal velocity.
(C) Kinetic energy and acceleration.
(D) Acceleration and momentum.

Ans. :

b. Acceleration and horizontal velocity.

16. If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then the value of $(\vec{a}_1 - \vec{a}_2).(2\vec{a}_1 + \vec{a}_2)$ is:

(A) 2
(B) $\frac{3}{2}$
(C) $\frac{1}{2}$
(D) 1

Ans. :

c. $\frac{1}{2}$

17. Two cars A and B move along a concentric circular path of radius r_A and r_B with velocities v_A and v_B maintaining constant distance, then $\frac{v_A}{v_B}$ is equal to:

(A) $\frac{r_B}{r_A}$
(B) $\frac{r_A}{r_B}$
(C) $\frac{r_A^2}{r_B^2}$
(D) $\frac{r_B^2}{r_A^2}$

Ans. :

b. $\frac{r_A}{r_B}$

18. A plane is inclined at an angle of 30° with horizontal. The magnitude of component of a vector $\vec{A} = -10\hat{k}$ perpendicular to this plane is (here z-direction is vertically upwards):

(A) $5\sqrt{2}$
(B) $5\sqrt{3}$
(C) 5
(D) 2.5

Ans. :

b. $5\sqrt{3}$

19. The angle between $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$ is

(A) 45° (B) 90° (C) –45° (D) 180°

Ans. :

b. 90°

Explanation:

Given $\vec{A} = \hat{i} + \hat{j}$

$\vec{B} = \hat{i} - \hat{j}$

$\vec{A}.\vec{B} = |A||B| \cos\theta$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$

$= \frac{(\hat{i}+\hat{j}).(\hat{i}-\hat{j})}{\sqrt{1^2+1^2} \times \sqrt{1^2+(-1)^2}} = \frac{1-1}{2} = 0$

$\Rightarrow \cos\theta = \cos 90$

$\therefore \theta = 90^\circ$. Hence, verifies the option (b).

20. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10kmh^{-1}. He finds that raindrops are hitting his head vertically. The actual speed of raindrops is:

(A) $20\mathrm{kmh}^{-1}$ (B) $10\sqrt{3}\mathrm{kmh}^{-1}$

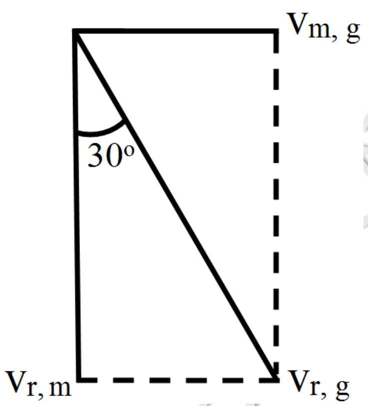
(C) $20\sqrt{3}\mathrm{kmh}^{-1}$ (D) $10\mathrm{kmh}^{-1}$

Ans. :

a. $20\mathrm{kmh}^{-1}$

Explanation:

When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.



Here, $V_{r,g}$ = Velocity of the rain with respect to the ground

$V_{m,g}$ = Velocity of the man with respect to the ground

and $V_{r,m}$ = Velocity of the rain with respect to the man.

Here, $V_{m,g}$ = 10 kmh^{-1}

$V_{r,\,g} = \frac{10}{\sin 30^\circ} = 20\mathrm{kmh}^{-1}$

21. A particle starts from origin at t = 0 with a velocity $5.0\hat{i}\ \mathrm{ms}^{-1}$ and moves in XY-plane under action of force which produces a constant acceleration of

$(3.0\hat{i}+2.0\hat{j})\ ms^{-2}$. What is the y-coordinate of the particle at the instant when its x-coordinate is 84m?

(A) 36m
(B) 24m
(C) 39m
(D) 18m

Ans. :

a. 36m

22. What is the position vector of a point mass moving on a circular path of radius of 10m with angular frequency of 2 rads^{-1} after $\frac{\pi}{8}$s? Initially the point was on Y-axis.

(A) $5.(\hat{i}+\hat{j})$
(B) $5\sqrt{2}(\hat{i}+\hat{j})$
(C) $\hat{i}+\hat{j}$
(D) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

Ans. :

b. $5\sqrt{2}(\hat{i}+\hat{j})$

23. The length of seconds hand of a watch is 1cm. The change in velocity of its tip in 15 seconds in cm/ s is:

(A) zero
(B) $\frac{x}{(30\sqrt{2})}$
(C) $\frac{\pi}{30}$
(D) $\frac{2\pi}{(30\sqrt{2})}$

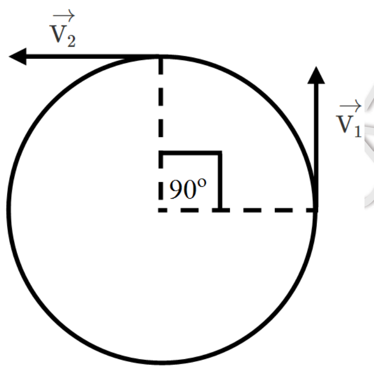
Ans. :

d. $\frac{2\pi}{(30\sqrt{2})}$

Explanation:

The angle described at the centre by the length of second hand of a watch in 15 second

$=90^{\circ}=\frac{\pi}{2}$ radians Figure.



Linear speed $v=r\omega=\frac{r\theta}{t}$

$=\frac{1\times\left(\frac{\pi}{2}\right)}{15}=\frac{\pi}{30}cm\ s^{-1}$

Magnitude of change in velocity in 15 sec;

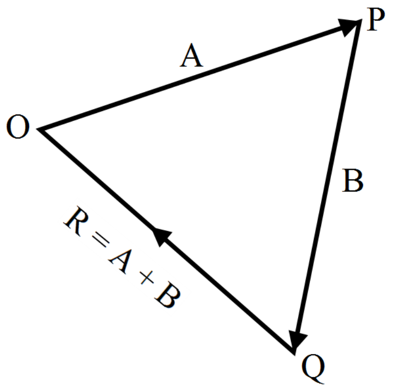
$|\Delta\vec{v}|=|\vec{v}_2-\vec{v}_1|=\sqrt{v_2^2-v_1^2}$

$=\sqrt{v^2+v^2}=\sqrt{2}v$

$=\sqrt{2}\frac{\pi}{30}=\frac{2\pi}{30\sqrt{2}}$

24. A and B are two inclined vectors. R is their sum. Choose the correct figure for the given description.

(A)



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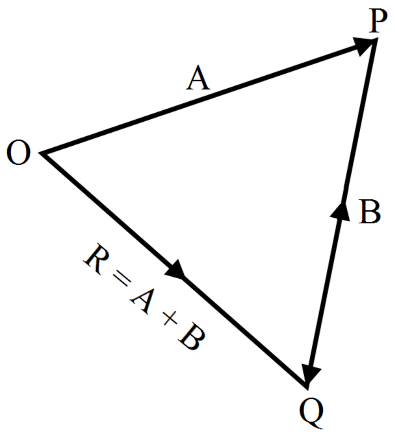
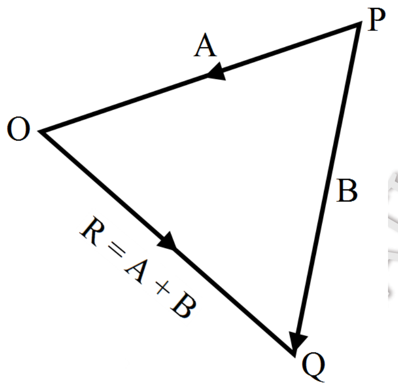
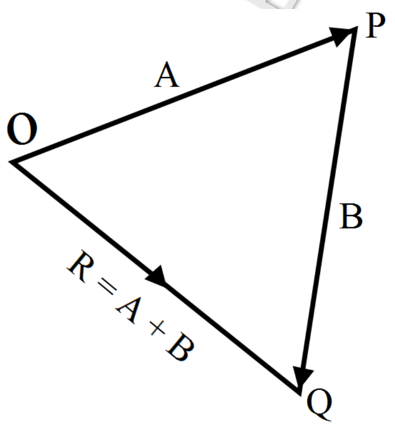


(C)

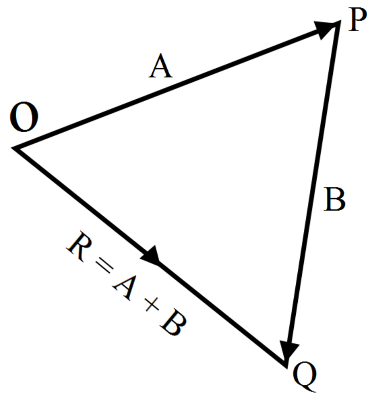


(D)

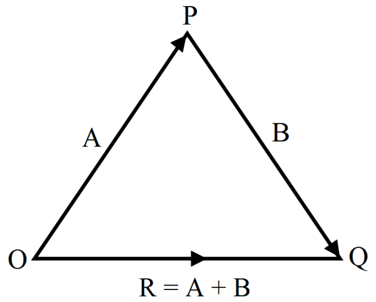


Ans. :

d.



Explanation:



25. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}$, $\vec{F}_2 = -5\hat{i} + 2\hat{k}$, and $\vec{F}_3 = 6\hat{i} - \hat{k}$ acting on a particle has a magnitude equal to 5 units, then the value of p is:

(A) -6

(B) -4

(C) 3

(D) 4

Ans. :

c. 3

26. The simple sum of two forces acting at a point is 16N and their sum is 8N and its direction is perpendicular to the smaller force, then the forces are:

(A) 6N and 10N

(B) 8N and 8N

(C) 4N and 12N

(D) 2N and 14N

Ans. :

a. 6N and 10N.

Explanation:

Here A + B = 16 ...(i)

$\sqrt{A^2 + B^2 + 2AB\cos\theta} = 8 \ldots \text{(ii)}$

and $\tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta}$

or $A + B\cos\theta = \frac{B\sin\theta}{\tan 90^\circ} = 0$

or $B\cos\theta = -A$ or $\cos\theta = \frac{-A}{B}$

From (ii), $A^2 + B^2 + 2AB\left(\frac{-A}{B}\right) = 64$

or $B^2 - A^2 = 64$

Solving (i) and (iii), we get

A = 6N and B = 10N.

27. A person moves 30m North, then 20m East then $30\sqrt{2}$ South-West. His displacement from the original position is:

(A) 14m South-West.

(B) 28m South.

(C) 10m West.

(D) 15m East.

Ans. :

c. 10m West.

Explanation:

Resolving displacement $30\sqrt{2}\ \mathrm{m}$ south-west in two rectangular components:

we have $30\sqrt{2}\cos 45^\circ = 30\sqrt{2} \times \frac{1}{\sqrt{2}} = 30\mathrm{m}$ towards south and

$30\sqrt{2} \times \sin 45^\circ = 30 \times \sqrt{2} \times \frac{1}{\sqrt{2}} = 30\mathrm{m}$ towards west.

The resultant of 30m north will neutralise the displacement of 30m south. Hence, the effective displacement is the resultant of 30m west and 20m east = 10m west.

28. If a unit vector is represented by $0.5\hat{\mathrm{i}} + 0.8\hat{\mathrm{j}} + \mathrm{c}\hat{\mathrm{k}}$, then the value of 'c' is:

(A) 1

(B) $\sqrt{0.11}$

(C) $\sqrt{0.01}$

(D) $\sqrt{0.39}$

Ans. :

b. $\sqrt{0.11}$

Explanation:

Here, $(0.5)^2 + (0.8)^2 + (c)^2 = 1$

or $\mathrm{c} = \sqrt{0.11}$

29. Angle that the vector $\vec{\mathrm{A}} = 2\hat{\mathrm{i}} + 2\hat{\mathrm{j}}$ makes with y-axis is:

(A) $\tan^{-1}\left(\frac{3}{2}\right)$

(B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\sin^{-1}\left(\frac{2}{3}\right)$

(D) $\cos^{-1}\left(\frac{3}{2}\right)$

Ans. :

b. $\tan^{-1}\left(\frac{2}{3}\right)$

Explanation:

As $\vec{\mathrm{A}} = 2\hat{\mathrm{i}} + 2\hat{\mathrm{j}}$, therefore $A_x = 2$ and $A_y = 3$. If θ is the angle which $\vec{\mathrm{A}}$ encloses with y-axis, then.

$\tan\theta = \frac{\mathrm{A_x}}{\mathrm{A_y}} = \frac{2}{3}$ or $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

30. The relation between the vectors A and -2A is that,

(A) Both have same magnitude.

(B) Both have same direction.

(C) They have opposite directions.

(D) None of the above.

Ans. :

c. They have opposite directions.

Explanation:

Multiplying a vector A by a negative number λ gives a vector $\lambda \mathrm{A},$ whose directions opposite to the direction of A and it's magnitude is $-\lambda$ times |A|.

31. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are

(A) Impulse, pressure and area.

(B) Impulse and area.

(C) Area and gravitational potential.

(D) Impulse and pressure.

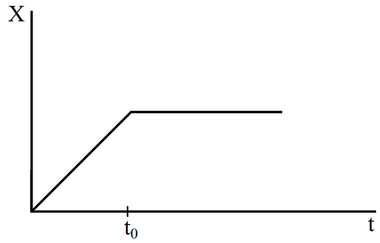
Ans. :

b. Impulse and area.

Explanation:

We know that impulse J = F. $\Delta \mathrm{t} = \Delta \mathrm{p},$ where F is force, At is time duration and Ap is change in momentum. As $\Delta \mathrm{p}$ is a vector quantity, hence impulse is also a vector quantity. Sometimes area can also be treated as vector direction of area vector is perpendicular to its plane.

32. Figure shows the displacement-time graph of a particle moving on the X-axis.



a. The particle is continuously going in positive x direction.
b. The particle is at rest.
c. The velocity increases up to a time to, and then becomes constant.
d. The particle moves at a constant velocity up to a time to, and then stops.

Ans. :

d. The particle moves at a constant velocity up to a time to, and then stops.

Explanation:

The slope of the x-t graph gives the velocity. In the graph, the slope is constant from t = 0 to t = f_0, so the velocity is constant. After t = t_0, the displacement is zero; i.e., the particle stops.

33. A particle moves along the X-axis as x = u(t - 2s) + a(t - 2s)^2.
a. The initial velocity of the particle is u.
b. The accelerati?n of the particle is a.
c. The acceleration of the particle is 2a.
d. At t = 2s particle is at the origin.

Ans. :

c. The acceleration of the particle is 2a.

d. At t = 2s particle is at the origin.

Explanation:

Initial velocity $= \left|\frac{dx}{dt}\right|_{t=0}$

$\frac{dx}{dt} = u + 2a(t - 2s)$

$\left|\frac{dx}{dt}\right|_{t=0} = u - 4as \neq u$

Acceleration $= \frac{d^2x}{dt^2} = 2a$

At t = 2s,

$x = u(2s - 2s) + a(2s - 2s)^2 = 0$ (origin)

34. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first?

a. The faster one.

b. The slower one.

c. Both will reach simultaneously.

d. Depends on the masses.

Ans. :

c. Both will reach simultaneously.

Explanation:

Because the downward acceleration and the initial velocity in downward direction of the two bullets are the same, they will take the same time to hit the ground and for a half projectile.

Time of flight $= T = \sqrt{\frac{2h}{g}}$

35. A person standing near the edge of the top of a building throws two balls A and B. The ball A is thrown vertically upward and B is thrown vertically downward with the same speed. The ball A hits the ground with a speed v_A and the ball B hits the ground with a speed v_B. We have:

a. $v_A > v_B$

b. $v_A < v_B$

c. $v_A = v_B$

d. The relation between v_A and v_B depends on height of the building above the ground.

Ans. :

c. $v_A = v_B$

Explanation:

Total energy of any particle $= \frac{1}{2}mv^2 + mgh$

Both the particles were at the same height and thrown with equal initial velocities, so their initial total energies are equal. By the law of conservation of energy, their final energies are equal.

At the ground, they are at the same height. So, their P.E. are also equal; this implies that their K.E. should also be equal. In other words, their final velocities are equal.

*** Answer The Following Questions In One Sentence.[1 Marks Each]** **[2]**

36. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:
    a. adding any two scalars.
    b. adding a scalar to a vector of the same dimensions.
    c. multiplying any vector by any scalar.
    d. multiplying any two scalars.
    e. adding any two vectors.
    f. adding a component of a vector to the same vector.

Ans. :
a. Yes, addition of two scalar quantities is meaningful only if they both represent the same physical quantity.
b. No, addition of a vector quantity with a scalar quantity is not meaningful.
c. Yes, scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.
d. Yes, scalar, irrespective of the physical quantity it represents, can be multiplied with another scalar having the same or different dimensions.
e. Yes, addition of two vector quantities is meaningful only if they both represent the same physical quantity.
f. Yes, component of a vector can be added to the same vector as they both have the same dimensions.

37. An aircraft executes a horizontal loop of radius 1.00km with a steady speed of 900km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Ans. : Radius of the loop, r = 1km = 1000m

Speed of the aircraft, $\mathrm{v} = 900\mathrm{km/h} = 900 \times \frac{5}{18} = 250\mathrm{m/s}$

Centripetal acceleration, $\mathrm{a_c} = \frac{\mathrm{v}^2}{\mathrm{r}}$

$= \frac{250^2}{1000} = 62.5\mathrm{m/s}^2$

Acceleration due to gravity, g = 9.8m/s^2

$\frac{\mathrm{a_c}}{\mathrm{g}} = \frac{62.5}{9.8} = 6.38$

$\mathrm{a_c} = 6.38\mathrm{g}$

*** Given Section consists of questions of 2 marks each.** **[36]**

38. Rain is falling vertically with a speed of $35ms^{-1}$. Winds starts blowing after sometime with a speed of $12ms^{-1}$ in east to west direction. In which direction

should a boy waiting at a bus stop hold his umbrella?

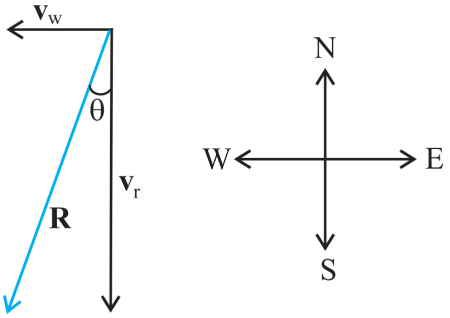


Fig. 3.7

Ans. : The velocity of the rain and the wind are represented by the vectors v_r and v_w in Fig. 3.7 and are in the direction specified by the problem. Using the rule of vector addition, we see that the resultant of v_r and v_w is R as shown in the figure. The magnitude of R is

$$R=\sqrt{v_r^2+v_w^2}=\sqrt{35^2+12^2}\,ms^{-1}=37ms^{-1}$$

The direction θ that R makes with the vertical is given by

$$\tan\theta=\frac{v_w}{v_r}=\frac{12}{35}=0.343$$

Or, $\quad \theta=\tan^{-1}(0.343)=19^{\circ}$

Therefore, the boy should hold his umbrella in the vertical plane at an angle of about 19° with the vertical towards the east.

39. A cricket ball is thrown at a speed of $28ms^{-1}$ in a direction 30 above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

Ans. : (a) The maximum height is given by

$$h_m=\frac{(v_o\sin\theta_o)^2}{2g}=\frac{(28\sin 30^{\circ})^2}{2(9.8)}m$$

$$=\frac{14\times 14}{2\times 9.8}=10.0m$$

(b) The time taken to return to the same level is

$$T_f=(2v_0\sin\theta_0)/g=(2\quad 28\quad \sin 30)/9.8$$

$$=28/9.8s=2.9s$$

(c) The distance from the thrower to the point where the ball returns to the same level is

$$R=\frac{(v_o^2\sin 2\theta_o)}{a}=\frac{28\times 28\times\sin 60^{\circ}}{98}=69m$$

40. An insect trapped in a circular groove of radius $12cm$ moves along the groove steadily and completes 7 revolutions in $100s$. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?

Ans. : This is an example of uniform circular motion. Here $R = 12cm$. The angular speed ω is given by

$$\omega = 2\pi/T = 2\pi \quad 7/100 = 0.44rad/s$$

The linear speed v is :

$$v = \omega R = 0.44s^{-1} \quad 12cm = 5.3cms^{-1}$$

The direction of velocity v is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is not a constant vector. However, the magnitude of acceleration is constant:

$$a = \omega^2 R = \left(0.44s^{-1}\right)^2 (12cm)$$

$$= 2.3cms^{-2}$$

41. Find the angle of projection for a projectile motion whose range R is n times the maximum height H.

Ans. : Given R = nH

$$\Rightarrow \frac{\mathrm{u}^2 \sin 2\theta}{\mathrm{g}} = \mathrm{n} \times \frac{\mathrm{u}^2 \sin^2 \theta}{2\mathrm{g}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{\mathrm{n}}\right)$$

42. A football is kicked 20m/ s at a projection angle of 45°. A receiver on the goal line 25m away in the direction of the kick runs the same instant to meet the ball. What must be his speed, if he has to catch the ball before it hits the ground?

Ans. : Given, u = 20m/ s, $\theta = 45^\circ$, d = 25m

Horizontal range is given by

$$\mathrm{R} = \frac{\mathrm{u}^2}{\mathrm{g}} \sin 2\theta = \frac{(20)^2}{9.8} \sin 2(45^\circ)$$

$$= \frac{400}{9.8} \times 1 = 40.82\mathrm{m}$$

Time of flight, $\mathrm{T} = \frac{2\mathrm{u} \sin \theta}{\mathrm{g}} = \frac{2 \times 20}{9.8} \sin 45^\circ$

$$= 2.886\mathrm{s}$$

The goal man is 25m away in the direction of the ball, so to catch the ball, he is to cover a distance

= 40.82 - 25 = 15.82m in time 2.886s.

$\therefore$ Velocity of the goal man to catch the ball

$$\mathrm{v} = \frac{15.82}{2.886} = 5.48\mathrm{m/\ s}$$

43. An aeroplane travelling at a speed of 5000km/ hr tilts at an angle of 30° as it makes a turn. What is the radius of the curve?

Ans. : $\tan \theta = \frac{\mathrm{v}^2}{\mathrm{rg}} = \left(\frac{1250}{9}\right)^2 \times \frac{1}{\mathrm{r}} \times \frac{1}{9.8}$

or $\mathrm{r} = \left(\frac{1250}{9}\right)^2 \frac{\sqrt{3}}{1} \times \frac{1}{9.8}$

$$\left[\because 5000 \text{ km/hr} = \frac{1250}{9} \text{ m/s}\right]$$

or, $r = 3.41 \times 10^3 m \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}} \right]$

44. Calculate the area of a parallelogram whose adjacent sides are given by the vectors. $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k};\ \vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}.$

Ans. : Area of the parallelogram $= |\vec{A} \times \vec{B}|$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -3 & 1 \end{vmatrix}$$

$= 11\hat{i} + 5\hat{j} - 7\hat{k}$

$\therefore\ |\vec{A} \times \vec{B}| = \sqrt{121 + 25 + 49}$

$= \sqrt{195}$ sq. units

45. Two forces whose magnitudes are in the ratio 3 : 5 give a resultant of 28N. If the angle of their inclination is 60°. Find the magnitude of each force.

Ans. : Let A and B be the two forces.

Then, A = 3x, B = 5x, R = 28N and $\theta = 60^\circ$

Thus, $\frac{A}{B} = \frac{3}{5}$

Now, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow\ 28 = \sqrt{9x^2 + 25x^2 + 30x^2 \cos 60^\circ} = 7x$

$\Rightarrow\ x = 4$

$\therefore$ Forces are A = 12N and B = 20N.

46. A cyclist has to bend a little inwards from his vertical position while turning. Why?

Ans. : By bending, a component of normal reaction of the ground is spared to provide him the necessary centripetal force for turning.

47. Two bodies are thrown with same velocities at angles α and $(90^\circ - \alpha)$ with the horizontal. What will be the ratio of (i) maximum heights attained by them (ii) their horizontal ranges?

Ans. :

i. $\frac{h_1}{h_2} = \frac{\frac{v_0^2 \sin^2 \alpha}{2g}}{\frac{v_0^2 \sin^2(90-\alpha)}{2g}}$

$= \frac{\sin^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha$

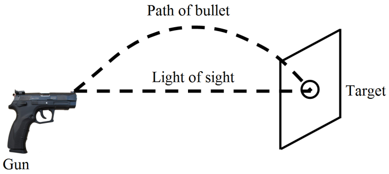
ii. Horizontal ranges in both the cases are same, i.e.,

$R_1 = R_2,\ \frac{R_1}{R_2} = 1$

48. A skilled gun man always keeps his gun slightly tilted above the line of sight while shooting. Why?

Ans. : When a bullet is fired from a gun with its barrel directed towards the target, it starts falling downwards on account of acceleration due to gravity.

Due to which the bullet hits below the target. Just to avoid it, the barrel of the gun is lined up little above the target, so that the bullet after travelling in parabolic path hits the distant target.



49. If $\vec{A} = (-2\hat{i} + 3\hat{j} - 4\hat{k})$ and $\vec{B} = (3\hat{i} - 4\hat{j} + 5\hat{k})$ find $\vec{A} \times \vec{B}$ and $\vec{A}.\vec{B}$

Ans. :

i. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -4 \\ 3 & -4 & 5 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} 3 & -4 \\ -4 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} -2 & -4 \\ 3 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} -2 & 3 \\ 3 & -4 \end{vmatrix}$

$= \hat{i}(15 - 16) - \hat{j}(-10 + 12) + \hat{k}(8 - 9)$

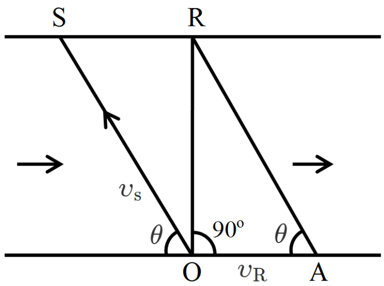
$= -\hat{i} - 2\hat{j} - \hat{k}$

ii. $\vec{A}.\vec{B} = (-2\hat{i} + 3\hat{j} - 4\hat{k}).(3\hat{i} - 4\hat{j} + 5\hat{k})$

$= -6 - 12 - 20 = -38$

50. A swimmer can swim with velocity of 10 km/ h. w.r.t. the water flowing in a river with velocity of 5 km/ h. In what direction should he swim to reach the point on the other bank just opposite to his starting point?

Ans. :



v_R = 5km/ h

v_S = 10km/ h

$\cos\theta = \frac{5}{10} = \frac{1}{2}$

$\theta = 60^\circ$

⇒ with the direction of river, the angle would be 120° to reach the opposite point.

51. A body is projected with a speed v at an angle θ with horizontal to have maximum range. What is the velocity at the highest point?

Ans. : For maximum range $\theta = 45^\circ$, velocity at the highest point $= v\cos 45^\circ = \frac{v}{\sqrt{2}}$.

52. Two forces 5kg-wt. and 10kg-wt. are acting with an inclination of 120° between them. Find the angle when the resultant makes with 10kg-wt.

Ans. : Given, A = 5kg-wt, B = 10kg-wt, $\theta = 120^{\circ}$ then $\beta = ?$

$\tan \beta = \frac{\mathrm{B} \sin \theta}{\mathrm{A}+\mathrm{B} \cos \theta} = \frac{10 \sin 120^{\circ}}{5+10 \cos 120^{\circ}}$

$= \frac{5 \sin 60^{\circ}}{10-5 \cos 60^{\circ}} = \frac{5 \times \frac{\sqrt{3}}{2}}{10-\frac{5}{2}} = \frac{1}{\sqrt{3}} = \tan 30^{\circ}$

$\therefore \ \beta = 30^{\circ}$

53. Two bombs of 20kg and 30kg are thrown from a cannon with the same velocity in the same direction. Which bomb will reach the ground first?

Ans. : Time of flight $\mathrm{T} = \frac{2\mathrm{u} \sin \theta}{\mathrm{g}}$. Since time of flight does not depend upon mass, both will reach simultaneously.

54. Determine that vector which when added to the resultant of $\mathrm{A} = 3\hat{\mathrm{i}} - 5\hat{\mathrm{j}} + 7\hat{\mathrm{k}}$ and $\mathrm{B} = 2\hat{\mathrm{i}} + 4\hat{\mathrm{j}} - 3\hat{\mathrm{k}}$ gives unit vector along y-direction.

Ans. : We are given, $\mathrm{A} = 3\hat{\mathrm{i}} - 5\hat{\mathrm{j}} + 7\hat{\mathrm{k}}$ and $\mathrm{B} = 2\hat{\mathrm{i}} + 4\hat{\mathrm{j}} - 3\hat{\mathrm{k}}$

Thus, the resultant vector is given by,

$\mathrm{R} = \mathrm{A} + \mathrm{B} = (3\hat{\mathrm{i}} - 5\hat{\mathrm{j}} + 7\hat{\mathrm{k}}) + (2\hat{\mathrm{i}} + 4\hat{\mathrm{j}} - 3\hat{\mathrm{k}})$

$= 5\hat{\mathrm{i}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$

But the unit vector along y-direction $= \hat{\mathrm{j}}$

$\therefore$ Required vector $= \hat{\mathrm{j}} - (5\hat{\mathrm{i}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}})$

$= -5\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - 4\hat{\mathrm{k}}$

55. Prove that the vectors $(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$ and $(2\hat{\mathrm{i}} - \hat{\mathrm{j}})$ are perpendicular to each other.

Ans. : $\vec{\mathrm{A}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$

$\vec{\mathrm{B}} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}}$

$\vec{\mathrm{A}}$ is perpendicular to $\vec{\mathrm{B}}$,

if $\vec{\mathrm{A}}.\vec{\mathrm{B}} = 0$

$\therefore \ (\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}).(2\hat{\mathrm{i}} - \hat{\mathrm{j}}) = 2 - 2 = 0$

Thus, $\vec{\mathrm{A}}$ is perpendicular to $\vec{\mathrm{B}}$.

*** Given Section consists of questions of 3 marks each.** **[141]**

56. A hiker stands on the edge of a cliff $490m$ above the ground and throws a stone horizontally with an initial speed of $15ms^{-1}$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g = 9.8ms^{-2}$).

Ans. : We choose the origin of the x^{-}, and y^{-} axis at the edge of the cliff and $t = 0s$ at the instant the stone is thrown. Choose the positive direction of x-axis to be along the initial velocity and the positive direction of y-axis to be the vertically upward direction. The x^{-}, and y^{-} components of the motion can be treated independently. The equations of motion are :

$x(t) = x_o + v_{ax} t$

Here,

$y(t) = y_o + v_{oy} t + (1/2) a_y t^2$

$x_o = y_o = 0, v_{oy} = 0, a_y = -g = -9.8 ms^{-2}$,

$v_{ox} = 15 ms^{-1}$.

The stone hits the ground when $y(t) = -490m$.

$-490m = -(1/2)(9.8)t^2$.

This gives $t = 10s$.

The velocity components are $v_x = v_{ox}$ and

$v_y = v_{oy} - gt$

so that when the stone hits the ground:

$v_{ax} = 15 ms^{-1}$

$v_{oy} = 0 - 9.8 \quad 10 = -98 ms^{-1}$

Therefore, the speed of the stone is

$\sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = 99 ms^{-1}$

57. A motorboat is racing towards north at $25 km/h$ and the water current in that region is $10 km/h$ in the direction of 60° east of south. Find the resultant velocity of the boat.

Ans. : The vector v_b representing the velocity of the motorboat and the vector v_c representing the water current are shown in Fig. 3.11 in directions specified by the problem. Using the parallelogram method of addition, the resultant R is obtained in the direction shown in the figure.

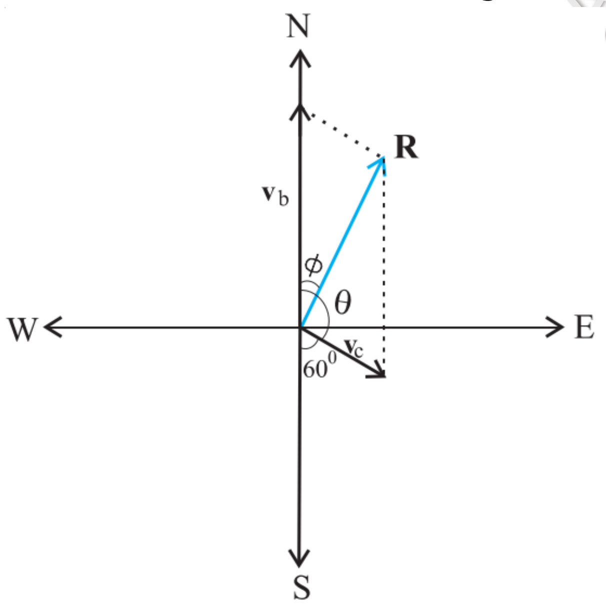


Fig. 3.11

We can obtain the magnitude of R using the Law of cosine :

$$R = \sqrt{v_b^2 + v_c^2 + 2 v_b v_c \cos 120^\circ}$$

$$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10(-1/2)} \cong 22 km/h$$

To obtain the direction, we apply the Law of sines

$\frac{R}{\sin\theta}=\frac{v_c}{\sin\phi}$ or, $\sin\phi=\frac{v_c}{R}\sin\theta$

$=\frac{10\times\sin 120^{\circ}}{21.8}=\frac{10\sqrt{3}}{2\times 21.8}\cong 0.397$

$\phi\cong 23.4^{\circ}$

58. A particle starts from origin at $t=0$ with a velocity $5.0\hat{i}m/s$ and moves in $x-y$ plane under action of a force which produces a constant acceleration of $(3.0\hat{i}+2.0\hat{j})m/s^2$. (a) What is the y-coordinate of the particle at the instant its x-coordinate is $84m$? (b) What is the speed of the particle at this time?

Ans. : From Eq. (3.34a) for $r_0=0$, the position of the particle is given by

$r(t)=v_0t+\frac{1}{2}at^2$

$=5.0\hat{i}t+(1/2)(3.0\hat{i}+2.0\hat{j})t^2$

$=\left(5.0t+1.5t^2\right)\hat{i}+1.0t^2\hat{j}$

Therefore,

$x(t)=5.0t+1.5t^2$

$y(t)=+1.0t^2$

Given

$x(t)=84m, t=\ ?$

$5.0t+1.5t^2=84\Rightarrow t=6s$

At $t=6s, y=1.0(6)^2=36.0m$

Now, the velocity $v=\frac{dr}{dt}=(5.0+3.0t)\hat{i}+2.0t\hat{j}$

At $t=6s, v=23.0\hat{i}+12.0\hat{j}$

$\text{speed }=|v|=\sqrt{23^2+12^2}\cong 26ms^{-1}$.

59. A man can swim with a speed of 4.0km/h in still water. How long does he take to cross a river 1.0km wide if the river flows steadily at 3.0km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Ans. : Speed of the man, v_m = 4 km/h

Width of the river = 1km

$\text{Time taken to cross the river}=\frac{\text{Width of the river}}{\text{Speed of the river}}$

$=\frac{1}{4}\text{h}=1\times\frac{60}{4}=15\text{min}$

Speed of the river, v_r = 3km/h

Distance covered with flow of the river = $v_r \times t$

$=3\times\frac{1}{4}=\frac{3}{4}\text{km}$

$=3\times\frac{1000}{4}$

= 750m.

60. A stone tied to the end of a string 80cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25s, what is the magnitude and

direction of acceleration of the stone?

Ans. : Length of the string, l = 80cm = 0.8m

Number of revolutions = 14

Time taken = 25s

Frequency, v $= \frac{= \text{Number of revolutions}}{\text{Time taken}}$

$= \frac{14}{25}\text{hz}$

Angular frequency, $\omega = 2\pi \text{v}$

$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}\ \text{rad s}^{-1}$

Centripetal acceleration, $\text{a}_\text{c} = \omega^2 \text{r}$

$= \left(\frac{88}{25}\right)^2 \times 0.8$

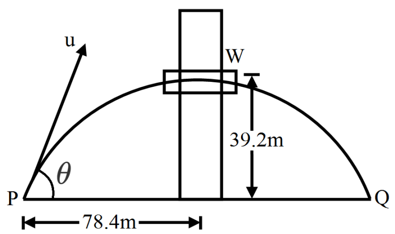
$= 9.91\text{ms}^{-2}$

The direction of centripetal acceleration is always directed along the string, toward the centre, at all points.

61. A boy stands at 78.4m from a building and throws a ball which just enters a window 39.2m above the ground. Calculate the velocity of projection of the ball.

Ans. : Consider a boy standing at P throw a ball with a velocity u at an angle with the horizontal which just enters window W.

As the boy is at 78.4m from the building and the ball just enters the window 39.2m above the ground.



$\therefore$ Maximum height, $\text{H} = \frac{\text{u}^2 \sin^2 \theta}{2\text{g}}$

$\Rightarrow\ 39.2 = \frac{\text{u}^2 \sin^2 \theta}{2\text{g}} \ldots \text{(i)}$

and horizontal range, $\text{R} = \frac{\text{u}^2 \sin 2\theta}{\text{g}}$

$\Rightarrow\ 2 \times 78.4 = \frac{\text{u}^2 \sin 2\theta}{\text{g}} \ldots \text{(ii)}$

Dividing eq. (i) by eq. (ii), we get

$\frac{\text{u}^2 \sin^2 \theta}{2\text{g}} \times \frac{\text{g}}{\text{u}^2 2 \sin\theta \cos\theta} = \frac{39.2}{2 \times 78.4}$

$\Rightarrow\ \frac{1}{4}\tan\theta = \frac{1}{4}$

$\Rightarrow\ \theta = 45^\circ$

Substituting $\theta = 45^\circ$ in eq. (ii), we get

$\frac{\text{u}^2 \sin 90^\circ}{9.8} = 2 \times 78.4$

$\Rightarrow\ \text{u} = \sqrt{2 \times 78.4 \times 9.8} = 39.2\text{m/ s}$

62. An aeroplane is flying in a horizontal direction with a velocity of 600km/ hr and at a height of 1960m. When it is vertically above the point A on the ground, a body is

dropped from it. The body strikes the ground at point B. Calculate the distance AB.

Ans. : Velocity of aeroplane in horizontal direction is,

$v_{ox} = 600 km/hr = 600 \times \frac{5}{18} ms^{-1}$

$= \frac{500}{3} ms^{-1}$

This velocity remains constant throughout the flight of the body.

$v_{oy} = 0$ and $y = h = 1960m$

Let t = the time taken by the body to reach the ground

Now, $y = v_{oy}t + \frac{1}{2}gt^2$

Here, $y = h = 1960m;\ v_{oy} = 0$ (initial vertical velocity)

$\therefore\ 1960 = \frac{1}{2} \times 9.8t^2$

$\therefore\ t = \sqrt{\frac{1960}{4.9}} s = \sqrt{400} s = 20s$

Distance travelled by the body in the horizontal direction

$= v_{ox}t = \frac{500}{3} \times 20 = \frac{10,000}{3}$

$= 3333m = 3.333km$

$\therefore\ AB = 3.333km.$

63. A person aims a gun at a bird from a point at a horizontal distance of 100m. If the gun can impart a speed of 500ms^{-1} to the bullet, at what height above the bird must he aim his gun in order to hit it?

Ans. : Horizontal distance, x = 100m

velocity, v = 500ms^{-1}

Time taken to travel this distance, $t = \frac{x}{v} = \frac{100}{500} = \frac{1}{5} s$

Vertical distance travelled by the bullet in time $\frac{1}{5} s$ is

$y = u_{oy}t + \frac{1}{2}gt^2$

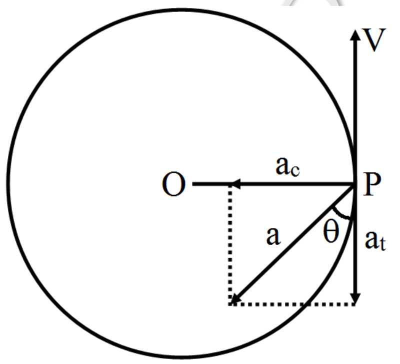
$= 0 + \frac{1}{2} \times 10 \times \frac{1}{25} = \frac{1}{5} m = 20cm.$

If he directly aims at the bird, the bullet will hit 20cm below the bird.

Therefore, the gun must be aimed at 20cm above the position of the bird.

64. A cyclist moving with a velocity of 7.5m s^{-1} approaches a U-turn of radius 80m. He applies brakes to slow down his speed at a rate of 0.5m s^{-2}. Calculate the acceleration of the cyclist on the turn.

Ans. :



v = 7.5m s^{-1}, r = 80m

Centripetal acceleration is

$a_c = \frac{v^2}{r} = \frac{7.5 \times 7.5}{80} = 0.7 \mathrm{m\,s^{-2}}$

When the cyclist applies brakes at P of the circular turn, then the tangential acceleration will act opposite to the velocity

i.e., a_t = 0.5m s^{-2}

$\therefore$ Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.7)^2 + (0.5)^2}$

= 0.86m s^{-2}

Let θ be the angle made by net acceleration with the velocity of the cylist, then

$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4$

$\therefore\ \theta = \tan^{-1}(1.4) = 54^\circ - 27'$

65. From the top of a tower 100m in height, a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25ms^{-1}. Find when and where the two balls will meet? g = 9.8ms^{-2}?

Ans. : $x = 0 + \frac{1}{2} \times 9.8t^2 = 4.9t^2 \ldots (i)$

$100 - x = 25t + \frac{1}{2} \times (-9.8)t^2 \ldots (ii)$

image

Solving equation (i) and (ii), we get

t = 4 second

x = 4.9 × 16 = 78.4m

66. A particle has a displacement of 12m towards east and 5m towards north and 6m vertically upwards. Find the magnitude of the sum of these displacements.

Ans. : The resultant displacement due to 12m towards east and 5m towards north lies in the plane of paper, and the angle between the displacements 90°, is given by

$R_1 = \sqrt{12^2 + 5^2} = 13\mathrm{m}$

Displacement 6m is vertically upward perpendicular to the plane of paper. Therefore, the angle between R_1 and 6m is 90°. The resultant of these two, R_2, will be $R_2 = \sqrt{13^2 + 6^2}$

$= \sqrt{205} = 14.32\mathrm{m}$

67. The range of a rifle bullet is 1000m, when e is the angle of projection. If the bullet is fired with the same angle from a car travelling at 36km/ h towards the target, show that the range will be increased by $142.9\sqrt{\tan\theta}\mathrm{m}$.

Ans. : Given, R = 1000m

$\therefore$ Horizontal range of the bullet fired at an angle θ is

$R = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow\ 1000 = \frac{u^2 2 \sin\theta \cos\theta}{g} \ldots (i)$

Bullet is fired from the car moving with 36km/ h

i.e.10m/ s, then horizontal component of the velocity of bullet $= u\sin\theta + 10$

Vertical component of the velocity of the bullet $= u\sin\theta$

Then, new range of the bullet is

$$R_1 = \frac{2}{g}(u\sin\theta)(u\cos\theta + 10)$$

$$= \frac{2}{g}u^2\sin\theta\cos\theta + \frac{20}{g}u\sin\theta$$

$$\Rightarrow \; R_1 = R + \frac{20}{g}u\sin\theta$$

$$\Rightarrow \; R_1 - R = \frac{20}{g}u\sin\theta \ldots \text{(ii)}$$

From Eq. (i), we have $u = \sqrt{\frac{1000\times g}{2\sin\theta\cos\theta}} \ldots \text{(iii)}$

Now substituting the value of u in Eq. (ii), we get

$$R_1 - R = \frac{20}{g}\sqrt{\frac{1000\times g}{2\sin\theta\cos\theta}}\sin\theta$$

$$= 20\sqrt{\frac{500\times\sin\theta}{g\cos\theta}}$$

$$= 20\sqrt{\frac{500}{9.8}\tan\theta} = 142.9\sqrt{\tan\theta}$$

68. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of next building which is of lower height than the first. If his speed is 9m/ s, the (horizontal) distance between the two building is 10m and the height difference is 9m, will he able to land on the next building? Substantiate your answer. Take g = 10m/ s^2.

Ans. : Time taken by the man to fall through a vertical height of 9m should be greater than time taken in horizontal distance for safe landing on the lower building.

Time taken to fall through a vertical height $t_1 = \sqrt{\frac{2h}{g}}$

$$= \sqrt{\frac{2\times 9}{10}} = 1.34 \text{ sec.}$$

Time taken to cover horizontal distance $t_2 = \frac{d}{v} = \frac{10}{9} = 1.11 \text{ sec}$

Thus, $t_1 > t_2$

Hence he will land safely on lower building.

69. A shell bursts on contact with the ground and the fragments fly in all directions with speeds upto 39.2m/ s. Show that a man 78.4m away is in danger for $4\sqrt{2}$ seconds.

Ans. : Given u = 39.2ms^{-1} and R = 78.4m

Horizontal Range,

$$R = \frac{u^2\sin 2\theta}{g}$$

or $\sin 2\theta = \frac{Rg}{u^2} = \frac{78.4\times 9.8}{(39.2)^2} = \frac{1}{2}$

or $\theta = 15^\circ \text{ or } 75^\circ$

It gives two times of flight, i.e.,

$$t_1 = \frac{2u\sin 75^\circ}{g}$$

$$t_2 = \frac{2u\sin 75^\circ}{g}$$

The man will be in danger for time (t_2 - t_1), i.e.,

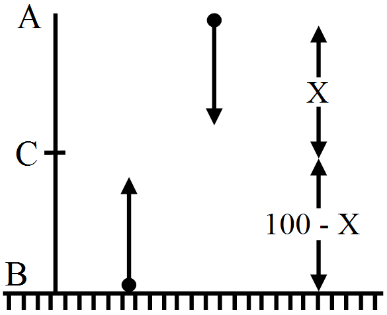
$$t_2 - t_1 = \frac{2u}{g}(\sin 75^\circ - \sin 15^\circ)$$

$= \frac{4u}{g} \sin 30^\circ \cos 45^\circ$
$= \frac{4 \times 39.2}{9.8} \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$
$= 2.83 \text{ seconds}$

70. From the top of a tower 100m in height, a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of $25 ms^{-1}$. Find when and where the two balls will meet? $g = 9.8 ms^{-2}$?

Ans. : $x = 0 + \frac{1}{2} \times 9.8t^2 = 4.9t^2 \ldots (i)$

$100 - x = 25t + \frac{1}{2} \times (-9.8)t^2 \ldots (ii)$



Solving equation (i) and (ii), we get

t = 4 second

x = 4.9 × 16 = 78.4m

71. Find the angle between force $\vec{F} = (3\vec{i} + 4\vec{j} - 5\vec{k})$ unit and displacement $\vec{d} = (5\vec{i} + 4\vec{j} + 3\vec{k})$ unit. Also find the projection of $\vec{F}$ on $\vec{d}$.

Ans. : $\vec{F} = (3\vec{i} + 4\vec{j} - 5\vec{k})$

$\vec{d} = (5\vec{i} + 4\vec{j} + 3\vec{k})$

$F = \sqrt{9 + 16 + 25} = \sqrt{50}$

$d = \sqrt{25 + 16 + 9} = \sqrt{50}$

$\vec{F}.\vec{d} = (3\hat{i} + 4\hat{j} - 5\hat{k}).(5\hat{i} + 4\hat{j} + 3\hat{k})$

$= 15 + 16 - 15 = 16$

Let Q be the angle between $\vec{F}$ and $\vec{d}$

$W = \vec{F}.\vec{d} = Fd\cos\theta$

$16 = \sqrt{50}\sqrt{50}\cos\theta$

$\frac{16}{50} = \cos\theta$

$\Rightarrow \theta = \cos^{-1}(0.32) = 71.3^\circ$

Unit vector along $\vec{d}$ is

$\hat{d} = \frac{5\hat{i} + 4\hat{j} + 3\hat{k}}{\sqrt{5^2 + 4^2 + 3^2}} = \frac{5\hat{i} + 4\hat{j} + 3\hat{k}}{\sqrt{50}}$

$\vec{F}.\vec{d} = \frac{16}{\sqrt{50}}$

Component vector of $\vec{F}$ along $\vec{d}$ is

$(\vec{F}.\vec{d})\hat{d} = \frac{16}{\sqrt{50}}\left(\frac{5\hat{i}+4\hat{j}+3\hat{k}}{\sqrt{50}}\right)$

$= 0.32(5\hat{i} + 4\hat{j} + 3\hat{k})$

72. Calculate the angular speed of the seconds hand of a clock. If the length of the seconds hand is 4cm, calculate the speed of the tip of the seconds hand.

Ans. : Seconds hand of a clock completes one rotation in 60s i.e.

$T = 60s,\ \theta = 2\pi\ \mathrm{rad}$

$\therefore$ Angular speed, $\omega = \frac{\theta}{T} = \frac{2\pi\,\mathrm{rad}}{60s}$

$= \frac{\pi}{30}\mathrm{rad\ s}^{-1}$

Length of the seconds hand, R = 4cm.

$\therefore$ Speed of the tip of second's hand is $v = \omega R = \frac{\pi}{30} \times 4 = \frac{2\pi}{15}\mathrm{cm\,s}^{-1}$.

73. Find a unit vector parallel to the vector $3\hat{i} + 7\hat{j} + 4\hat{k}$.

Ans. : Let $3\hat{i} + 7\hat{j} + 4\hat{k} = \vec{a}$.

$|\vec{a}| = \sqrt{3^2 + 7^2 + 4^2}$

$= \sqrt{9 + 49 + 16} = \sqrt{74}$

Using vector in the direction of

$\vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3\hat{i}+7\hat{j}+4\hat{k}}{\sqrt{74}}$

74. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. What will the percentage increase in the horizontal range?

Ans. : As, maximum height, $H = \frac{u^2}{2g}\sin^2\theta$

Consider ΔH be the increase in H when u changes by Δu, it can be obtained by differentiating the above equation, we get

$\Delta H = \frac{2u\Delta u \sin^2\theta}{2g} = \frac{2\Delta u}{u}H$

$\Rightarrow\ \frac{\Delta H}{H} = \frac{2\Delta u}{u}$

Given, % increase in H is 10%, so

$\frac{\Delta H}{H} = \frac{10}{100} = 0.1 \Rightarrow\ \frac{2\Delta u}{u} = 0.1$

As, $R = \frac{u^2 \sin 2\theta}{g}$

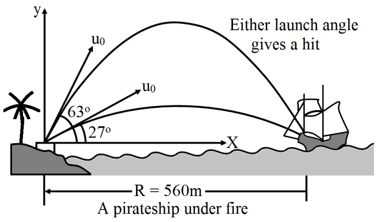
$\because\ \Delta R = \frac{2u\Delta u}{g}\sin 2\theta$

$\Rightarrow\ \frac{\Delta R}{R} = \frac{2\Delta u}{u} = 0.1$

$\therefore$ % increase in horizontal range $= \frac{\Delta R}{R} \times 100$

$= 0.1 \times 100 = 10\%$

75. Figure shows a pirateship 560m from a fort defending a harbour entrance. A defence cannon, located at sea level, fires balls at initial speed, u_0 = 82m/ s.



i. At what angle, θ_0 from the horizontal must a ball be fired to hit the ship?
ii. What is the maximum range of the cannon balls?

Ans. :

i. A fired cannon ball is a projectile and we want an equation that relates the launch angle, to the ball horizontal displacement i.e. range as it moves from the cannon to the ship.

$\therefore\ \theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{gR}{u_0^2}\right)$

$= \frac{1}{2}\sin^{-1}\left(\frac{9.8\times560}{(82)^2}\right)$

If one angle is 27°, then other angle $(90^\circ - \theta_0)$ is

$= 90^\circ - 27^\circ = 63^\circ$

ii. Maximum range at $\theta_0 = 45^\circ$

$\therefore\ R = \frac{u^2}{g}\sin 2\theta_0$

$= \frac{(82)^2}{9.8} \times \sin 90^\circ$

$= 686m$

76. An accelerating train is passing over a high bridge. A stone is dropped from the train at an instant when its speed is 10m/ s and acceleration is 1m/ s^2. Find the horizontal and vertical components of the velocity and acceleration of the stone one second after it is dropped. Take g = 10m/ s^2.

Ans. : Horizontal acceleration of the train is not carried by the stone. Horizontal velocity of the stone will remain constant, during the fall of the stone.

	Horizontal component	**Vertical component**
Velocity	10m/ s	10m/ s
Acceleration	0m/ s^2	10m/ s^2

77. A man can jump on moon about six times as high as on the earth. Why?

Ans. : We know that value of g on surface of moon is nearly $\frac{1}{6}$th of its value at earth. We also know that maximum height which a man can cover is given by $h = \frac{u^2\sin^2\theta}{2g}$

i.e., $h \propto \frac{1}{g}$

Therefore, $\frac{h_{moon}}{h_{earth}} = \frac{g_{earth}}{g_{moon}} = \frac{g}{\frac{g}{6}} = 6$

or $h_{moon} = 6h_{earth}$.

78. Show that vectors $A = 2\hat{i} - 3\hat{j} - \hat{k}$ and $B = -6\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

Ans. : The given vectors are

$A = 2\hat{i} - 3\hat{j} - \hat{k}$

and $B = -6\hat{i} + 9\hat{j} + 3\hat{k}$

Then, the vecotrs are parallel, if A × B = 0

$$\therefore \; A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$

$\hat{i}(-9+9) - \hat{j}(6-6) + \hat{k}(18-18) = 0$

But |A × B| = 0

$AB\sin\theta = 0 \;\; [\because A \neq 0 \text{ and } B \neq 0]$

$\therefore \;\; \sin\theta = 0 \text{ or } \theta = 0$

Hence, the vectors A and B are parallel.

79. An aeroplane is flying in a horizontal direction with a velocity of 600km/ h and at a height of 1960m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.

Ans. : Velocity of the aeroplane in the horizontal direction is

$u_{0y} = 600\text{km/ h} = 600 \times \frac{5}{18} = \frac{500}{3}\text{m/ s}$

Velocity remains constant throughout the flight of the body.

u_{0y} = 0 and y = h = 1960m

Let t = time taken by the body to reach the ground

Now, $y = u_{0y}t + \frac{1}{2}gt^2$

Here, $y = h = 1960\text{m}, \; u_{0y} = 0$

$\therefore \;\; 1960 = \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow \;\; t = \sqrt{\frac{1960}{4.9}} = \sqrt{400} = 20\text{s}$

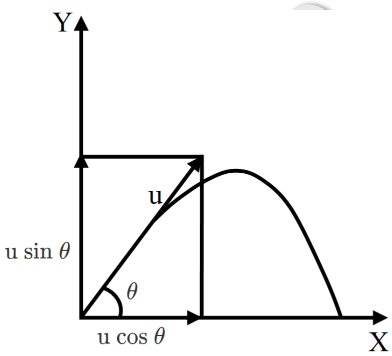
Distance travelled by the body in the horizontal direction,

$AB = x = v_{ax}t = \frac{500}{3} \times 20$

$= \frac{10000}{3} = 3333\text{m} = 3.33\text{km}$

80. Prove that the path of a projectile is a parabola.

Ans. : Consider a projectile thrown at an angle θ with a velocity u. The components of velocity horizontal and vertical are $u\cos\theta$ and $u\sin\theta$. After time t, the horizontal displacement $x = u\cos\theta t$



The vertical displacement,

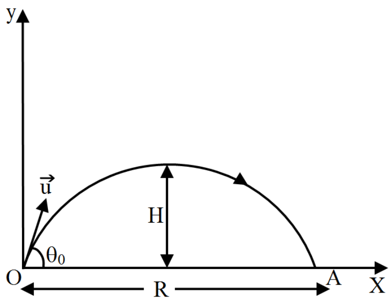
$y = u \sin\theta \, t - \frac{1}{2} g t^2$

$\therefore \; y = u \sin\theta . \left(\frac{x}{u \cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{u \cos\theta}\right)^2$

$y \propto x^2$, the path of a projectile is a parabola.

81. Show that the projection angle θ_0 for a projectile launched from the origin is given by, $\theta_0 = \tan^{-1}\left(\frac{4H}{R}\right)$ where, H is the maximum height attained by the projectile and R is the range of the projectile.

Ans. :



The path followed by a proiectile prolected at an angle θ_0 with velocity $\vec{u}$ is shown in figure. The maximum height attained by the projectile is given by

$H = \frac{u^2 \sin^2\theta_0}{2g} \ldots (i)$

The range of the projectile is given by

$R = \frac{u^2 \sin 2\theta_0}{g}$

$= \frac{2u^2 \sin\theta_0 \cos\theta_0}{g} \ldots (ii)$

Dividing eq. (i) by eq. (ii), we get

$\tan\theta_0 = \frac{4H}{R} \Rightarrow \; \theta_0 = \tan^{-1}\left(\frac{4H}{R}\right).$

82. The position of a particle is given by: $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4\hat{k}$ m where t is in seconds, r is in metres and the coefficients have the proper units.
   a. Find the velocity v and acceleration a.
   b. What is the magnitude of velocity of the particle at t = 2s?

Ans. : $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4\hat{k}$ m

a. $\vec{v}$ = velocity $= \frac{d\vec{r}}{dt} = 3.0\hat{i} - 4.0t\,\hat{j}$ m/s

$\vec{a}$ = acceleration $= \frac{d\vec{v}}{dt} = -4.0\,\hat{j}$ m/s^2

b. Magnitude of velocity at t = 2s

$\vec{v} = 3.0\hat{i} - 8.0\hat{j}$

$|\vec{v}| = \sqrt{(3)^2 + (-8)^2}$

$= \sqrt{9 + 64} = \sqrt{73}$ m/s

83. Determine a unit vector which is perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

Ans. : Unit vector perpendicular to both,

$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$

is given by $\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$$= \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}[2 - (-1)] - \hat{j}(4 - 1) + \hat{k}(-2 - 1)$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

Unit vector is $\hat{n} = \frac{3\hat{i}-3\hat{j}-3\hat{k}}{\sqrt{9+9+9}} = \frac{3\hat{i}-3\hat{j}-3\hat{k}}{\sqrt{27}}$

84. A bullet fired at an angle of 30° with the horizontal hits the ground 3km away. By adjusting its angle of projection, can one hope to hit a target 5km away? Assume the muzzle speed to be fixed, and neglect air resistance. (Take g = $10ms^{-2}$)

Ans. : Horizontal range R = 3000m

If u and θ denote the velocity of the bullet, and the angle made by the bullet with the horizontal, we have

$R = \frac{u^2 \sin 2\theta}{g}$

or $3000 = \frac{u^2 \sin 60}{10}$

or $u^2 = \frac{3000\times10\times2}{\sqrt{3}} = 34641ms^{-1}$

Solving, we get $u = 186.12ms^{-1}$

for maximum range, $\theta = 45^\circ$

$\sin 2\theta = \sin 90^\circ = 1$

$R_{max} = \frac{u^2}{g} = \frac{34641}{10} = 3464.1m$

$= 3.46\ km$

Thus, for the fixed muzzle speed, the maximum range of bullet is 3.46km. Therefore, it cannot hit a target 5km away.

85. The sum of the magnitude of two forces acting at a point is 18N and the magnitude of their resultant is 12N. If the resultant is at 90° with the force of smaller magnitude, what are the magnitude of forces?

Ans. : Let A and B be the two forces acting at a point and θ be the angle between them. Then

$A + B = 18 \ldots (i)$

And $A^2 + B^2 + 2AB\cos\theta = 12^2 = 144 \ldots (ii)$

If A is the smaller force, then as per question

$\tan 90^\circ = \frac{B\sin\theta}{A+B\cos\theta}$

$\infty = \frac{B\sin\theta}{A+B\cos\theta}$

$A + B\cos\theta = 0$

$\cos\theta = -\frac{A}{B}$

From (ii), $A^2 + B^2 + 2AB\left(-\frac{A}{B}\right) = 144$

$$B^2 - A^2 = 144$$

$$(B - A)(B + A) = 144$$

$$(B - A) = \frac{144}{(B+A)} = \frac{144}{18} = 8 \ldots (iii)$$

Solving (i) and (ii)

A = 5N and B = 13N

86. A fighter jet makes a loop of 1000m with a speed of 250m s^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.

Ans. : Here r = 1000m

v = 250m s^{-1}

Centripetal acceleration is given by

$$a = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{m s}^{-2}$$

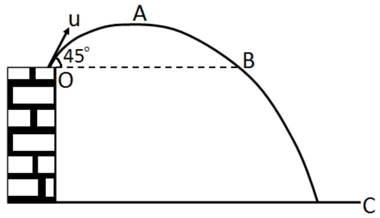
Acceleration due to gravity, g = 9.8m s^{-2}

$$\therefore \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = 6.4$$

87. A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have. Explain?

a. greatest speed.
b. smallest speed.
c. greatest acceleration?

Ans. :



In this problem total mechanical energy of the ball is conserved. As the ball is projected from point O, and covering the path OABC.

At point A it has both kinetic and potential energy.

But at point C it have only kinetic energy, (keeping the ground as reference where PE is zero.)

a. At point B, it will gain the same speed u and after that speed increases and will be maximum just before reaching C.
b. During upward journey from O to A speed decreases and smallest speed attained by it is at the highest point, i.e., at point A.
c. Acceleration is always constant throughout the journey and is vertically downward equal to g.

88. Earth also moves in circular orbit around sun once every year with on orbital radius of 1.5×10^{11}m. What is the acceleration of earth (or any object on the surface of the

earth) towards the centre of the sun? How does this acceleration compare with g = 9.8m/ s^2? $\left(\text{Hint: acceleration}\frac{V^2}{R} = \frac{4\pi^2 R}{T^2}\right)$

Ans. :

Orbital radius of the earth around the sun (R) = 1.5 × 10^{11}m

Time period = 1 year = 365 days = 365 × 24 × 60 × 60s = 3.15 × 10^{7}s

Centripetal acceleration

$$(a_c) = R\omega^2 = \frac{4\pi^2 R}{T^2} = \frac{4\times\left(\frac{22}{7}\right)^2\times1.5\times10^{11}}{(3.15\times10^7)^2}$$

$$= 5.97 \times 10^{-3} \mathrm{m/s}^2$$

$$\frac{a_C}{g} = \frac{5.97\times10^{-3}}{9.8} = \frac{1}{1642}$$

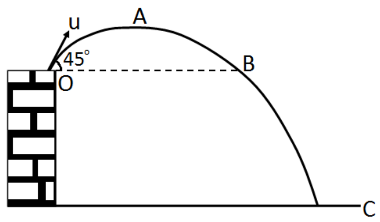
89. A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have. Explain?

a. greatest speed.

b. smallest speed.

c. greatest acceleration?

Ans. :



In this problem total mechanical energy of the ball is conserved. As the ball is projected from point O, and covering the path OABC.

At point A it has both kinetic and potential energy.

But at point C it have only kinetic energy, (keeping the ground as reference where PE is zero.)

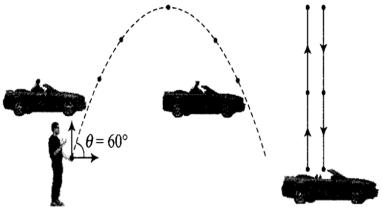
a. At point B, it will gain the same speed u and after that speed increases and will be maximum just before reaching C.

b. During upward journey from O to A speed decreases and smallest speed attained by it is at the highest point, i.e., at point A.

c. Acceleration is always constant throughout the journey and is vertically downward equal to g.

90. A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10m/ s (36km/ h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/ h). Give explanation to support your diagram.

Ans. : The situation is shown in the below diagram.



According to the problem the boy standing on ground throws the ball at an angle of 60° with horizontal at a speed of 10m/ s.

$\therefore$ Horizontal component of velocity, $u_x = 10\cos\theta$

$$u_x = (10\text{m/s})\cos 60^\circ = 10 \times \frac{1}{2} = 5\text{m/s}$$

Vertical component of velocity, $u_y = 10\sin\theta$

$$u_y = (10\text{m/s})\sin 60^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{m/s}$$

Speed of the car = 18km/ h = 5m/ s

As horizontal speed of ball and car is same, hence relative velocity of ball w.r.t car in the horizontal direction will be zero.

Only vertical motion of the ball will be observed by the boy in the car, as shown in above diagram.

91. A ball is projected vertically upward with a speed of 50m/s. Find:
   a. The maximum height.
   b. The time to reach the maximum height.
   c. The speed at half the maximum height. Take g = 10m/s^2.

Ans. : u = 50m/s, g = -10m/s^2 when moving upward, v = 0 (at highest point).

a. $S = \frac{v^2 - u^2}{2a} = \frac{0-15}{2(-10)} = 125\text{m}$

   maximum height reached = 125m

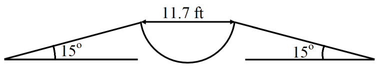
b. $t = \frac{(v-u)}{a} = \frac{(0-50)}{-10} = 5 \text{ sec.}$

c. $s' = \frac{125}{2} = 62.5\text{m}, u = 50\text{m/s}, a = -10\text{m/s}^2,$

   $v^2 - u^2 = 2as$

   $\Rightarrow v = \sqrt{(u^2 + 2as)} = \sqrt{50^2 + (-10)(62.5)} = 35\text{m/s}.$

92. Figure shows a 11.7ft wide ditch with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motorbike be moving on the road so that it safely crosses the ditch? Assume that the length of the bike is 5ft, and it leaves the road when the front part runs out of the approach road.



Ans. : Horizontal range X = 11.7 + 5 = 16.7ft covered by te bike.

g = 9.8m/s^2 = 32.2ft/s^2.

y = x

$$\tan\theta - \frac{gx^2 \sec^2\theta}{2u^2}$$

To find, minimum speed for just crossing, the ditch

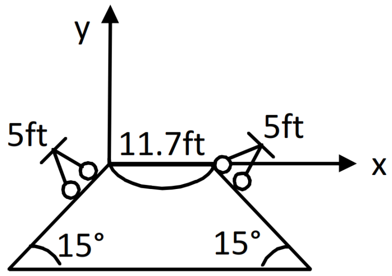
y = 0 ($\because$ A is on the x axis)

$$\Rightarrow x\tan\theta = \frac{gx^2\sec^2\theta}{2u^2}$$

$$\Rightarrow u^2 = \frac{gx^2 \sec^2\theta}{2x\tan\theta} = \frac{gx}{2\sin\theta\cos\theta} = \frac{gx}{\sin 2\theta}$$

$$\Rightarrow u = \sqrt{\frac{(32.2)(16.7)}{\frac{1}{2}}}\ \left(\because \sin 30^\circ = \frac{1}{2}\right)$$

⇒ u = 32.79ft/s = 32ft/s.



93. A ball is thrown at a speed of 40m/s at an angle of 60° with the horizontal. Find:
    a. The maximum height reached.
    b. The range of the ball. Take g = 10m/s^2.

Ans. : u = 40m/s, a = g= 9.8m/s^2, θ = 60° Angle of projection.

a. Maximum height h $= \frac{u^2\sin^2\theta}{2g} = \frac{40^2(\sin 60^\circ)^2}{2\times 10} = 60\text{m}$

b. Horizontal range X $= \frac{(u^2 \sin 2\theta)}{g}$

$$= \frac{(40^2 \sin 2(60^\circ))}{10} = 80\sqrt{3}\text{m}.$$

94. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

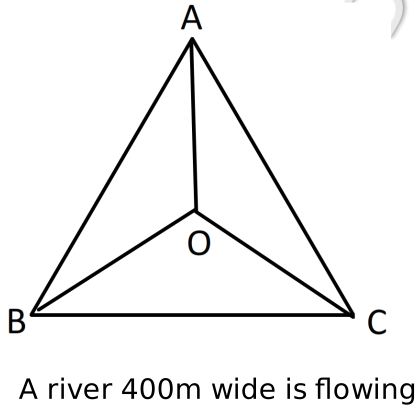
Ans. : The particles meet at the centroid O of the triangle. At any instant the particles will form an equilateral $\triangle ABC$ with the same centroid.

Consider the motion of particle A. At any instant its velocity makes angle 30°. This component is the rate of decrease of the distance AO.

Initially AO $= \frac{2}{3}\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{3}}$

Therefore, the time taken for AO to become zero.

$$= \frac{\frac{a}{\sqrt{3}}}{v\cos 30^\circ} = \frac{2a}{\sqrt{3}v\times\sqrt{3}} = \frac{2a}{3v}$$



95. A river 400m wide is flowing at a rate of 2.0m/s. A boat is sailing at a velocity of 10m/s with respect to the water, in a direction perpendicular to the river.
    a. Find the time taken by the boat to reach the opposite bank.

b. How far from the point directly opposite to the starting point does the boat reach the opposite bank?

Ans. :

a. Here the boat moves with the resultant velocity R. But the vertical component 10m/s takes him to the opposite shore.

$\tan\theta = \frac{2}{10} = \frac{1}{5}$

Velocity = 10m/s

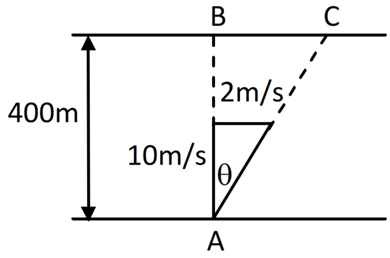
distance = 400m

$\text{Time} = \frac{400}{10} = 40\,\text{sec.}$

b. The boat will reach at point C.

In $\triangle \text{ABC}, \tan\theta = \frac{\text{BC}}{\text{AB}} = \frac{\text{BC}}{400} = \frac{1}{5}$

$\Rightarrow \text{BC} = \frac{400}{5} = 80\text{m}$

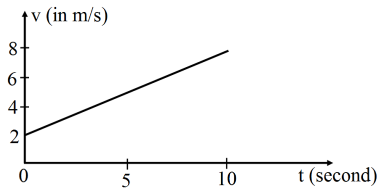


96. Figure shows the graph of velocity versus time for a particle going along the X-axis. Find:

a. The acceleration.

b. The distance travelled in 0 to 10s.

c. The displacement in 0 to 10s.



Ans. :

a. Initial velocity u = 2m/s.

final velocity v = 8m/s

time = 10 sec,

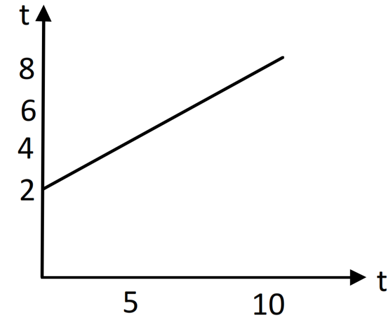
acceleration $= \frac{\text{v}-\text{u}}{\text{ta}} = \frac{8-2}{10} = 0.6\text{m/s}^2$

b. $v^2 - u^2 = 2aS$

$\Rightarrow \text{Distance S} = \frac{\text{v}^2-\text{u}^2}{2\text{a}} = \frac{8^2-2^2}{2\times0.6} = 50\text{m.}$

c. Displacement is same as distance travelled.

Displacement = 50m.



97. A man is sitting on the shore of a river. He is in the line of a 1.0m long boat and is 5.5m away from the centre of the boat. He wishes to throw an apple into the boat. If he can throw the apple only with a speed of 10m/s, find the minimum and maximum angles of projection for successful shot. Assume that the point of projection and the edge of the boat are in the same horizontal level.

Ans. : When the apple just touches the end B of the boat.

x = 5m, u = 10m/s, g = 10m/s^2, θ = ?

$\mathrm{x} = \frac{\mathrm{u}^2 \sin 2\theta}{\mathrm{g}}$

$\Rightarrow 5 = \frac{10^2 \sin 2\theta}{10}$

$\Rightarrow 5 = 10 \sin 2\theta$

$\Rightarrow \sin 2\theta = \frac{1}{2}$

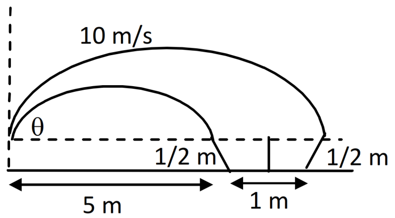
$\Rightarrow \sin 30^\circ$ or $\sin 150^\circ$

$\Rightarrow \theta = 15^\circ$ or 75°

Similarly for end C, x = 6m

Then $2\theta_1 = \sin^{-1}\left(\frac{\mathrm{gx}}{\mathrm{u}^2}\right) = \sin^{-1}(0.6) = 182^\circ$ or 71°.

So, for a successful shot, θ may very from 15° to 18° or 71° to 75°.



98. A person standing on the top of a cliff 171ft high has to throw a packet to his friend standing on the ground 228ft horizontally away. If he throws the packet directly aiming at the friend with a speed of 15.0ft/s, how short will the packet fall?

Ans. : $\tan\theta = \frac{171}{228}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{171}{228}\right)$

The motion of projectile (i.e. the packed) is from A. Taken reference axis at A.

$\therefore \theta = -37^\circ$ as u is below x-axis.

u = 15ft/s, g = 32.2ft/s^2, y = -171ft

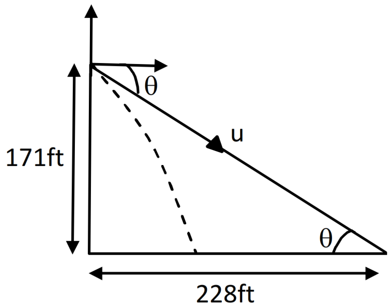
$\mathrm{y} = \mathrm{x}\tan\theta - \frac{\mathrm{x}^2\mathrm{g}\sec^2\theta}{2\mathrm{u}^2}$

$\therefore -171 = -\mathrm{x}(0.7536) - \frac{\mathrm{x}^2\mathrm{g}(1.568)}{2(225)}$

$$\Rightarrow 0.1125x^2 + 0.7536x - 171 = 0$$
$$= 35.78ft$$

Horizontal range covered by the packet is 35.78ft.

So, the packet will fall 228 - 35.78 = 192ft short of his friend.



99. A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

Ans. : For every ball, u = 0, a = g = 9.8m/s^2

∴ 4th ball move for 2 sec, 5th ball 1 sec and 3rd ball 3 sec when 6th ball is being dropped.

For 3rd ball t = 3 sec

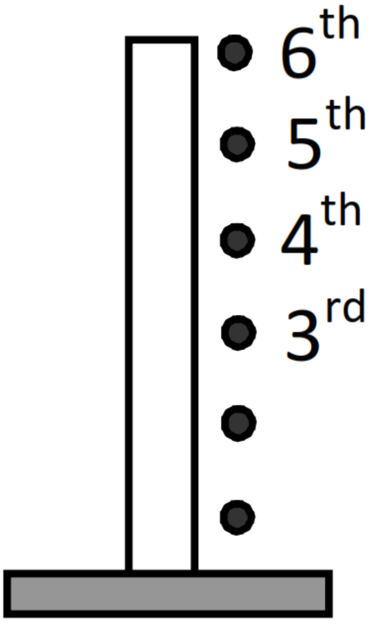
$S_3 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.8)3^2 = 4.9m$ below the top.

For 4th ball, t = 2 sec

$S_2 = 0 + \frac{1}{2}\ gt^2 = \frac{1}{2}(9.8)2^2 = 19.6m$ below the top (u = 0)

For 5th ball, t = 1 sec

$S_3 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.8)t^2 = 4.98m$ below the top.



100. A ball is dropped from a height. If it takes 0.200s to cross the last 6.00m before hitting the ground, find the height from which it was dropped. Take g = 10m/s^2.

Ans. : For last 6m distance travelled s = 6m, u = ?

t = 0.2 sec, a = g = 9.8m/s^2

$S = ut + \frac{1}{2}at^2$

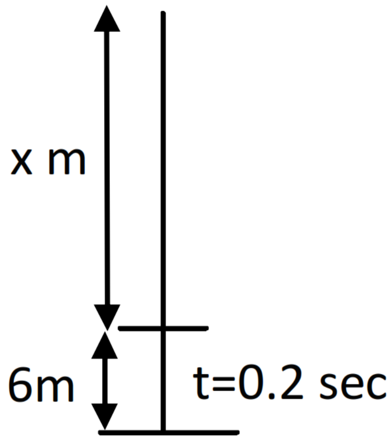
$\Rightarrow 6 = u(0.2) + 4.9 \times 0.04$

$\Rightarrow u = \frac{5.8}{0.2} = 29m/s.$

For distance x, u = 0, v = 29m/s, a = g = 9.8m/s^2

$S = \frac{v^2 - u^2}{2a} = \frac{29^2 - 0^2}{2 \times 9.8} = 42.05m$

Total distance = 42.05 + 6 = 48.05 = 48m.



101. A man has to go 50m due north, 40m due east and 20m due south to reach a field.

   a. What distance he has to walk to reach the field?

   b. What is his displacement from his house to the field?

Ans. :

   a. Distance travelled = 50 + 40 + 20 = 110m

   b. AF = AB - BF = AB - DC = 50 - 20 = 30m

His displacement is AD

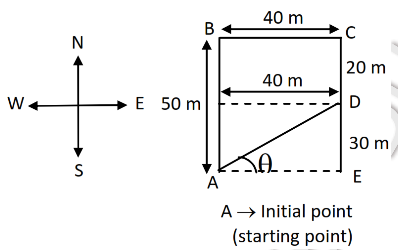
$AD = \sqrt{AF^2 - DF^2} = \sqrt{30^2 + 40^2} = 50m$

In $\triangle AED$

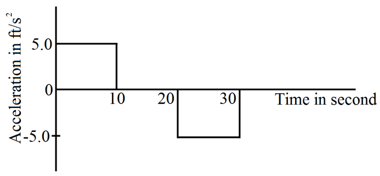
$\Rightarrow \tan\theta = \frac{DE}{AE} = \frac{30}{40} = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

His displacement from his house to the field is 50m, $\tan^{-1}\left(\frac{3}{4}\right)$ north to east.



102. The acceleration of a cart started at t = 0, varies with time as shown in figure. Find the distance travelled in 30 seconds and draw the position-time graph.



Ans. : In 1st seconds,

$S_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5 \times 10^2 = 250ft.$

At t = 10s,

v = u + at = 0 + 5 × 10 = 50ft/s

∴ From 10 to 20 seconds $(\triangle t = 20 - 10 = 10 \text{ sec})$ moves with uniform velocity 50ft/sec,

Distance S_2 = 50 × 10 = 500ft

Between 20 sec to 30 sec acceleration is constant i.e. $-5ft/s^2$. At 20 sec velocity is 50ft/sec.

t = 30 - 20 = 10s

$S_3 = ut + \frac{1}{2}at^2 = 50 \times 10 + \frac{1}{2}(-5)10^2$

$S_3 = 500 - 250 = 250ft$

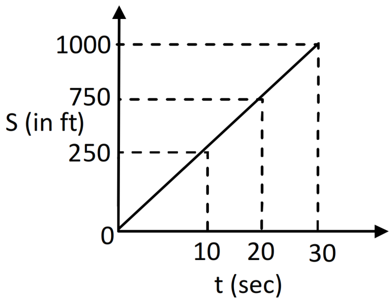
Total distance travelled is 30s:

$S_1 + S_2 + S_3$

= 250 + 500 + 250

= 1000ft

The position-time graph:



*** Given Section consists of questions of 5 marks each. [140]**

103. The ceiling of a long hall is 25m high. What is the maximum horizontal distance that a ball thrown with a speed of $40ms^{-1}$ can go without hitting the ceiling of the hall?

Ans. : Given:

Speed of the ball, u = 40m/s

Maximum height, h = 25m

In projectile motion, the maximum height reached by a body projected at an angle θ, is given by the relation:

$h = (u^2 \sin^2 \theta)$

$25 = \frac{40^2 \sin^2 \theta}{2 \times 9.8}$

$\sin^2 \theta = 0.30625$

$\sin \theta = 0.5534$

$\theta = \sin^{-1}(0.5534) = 33.60^\circ$

Horizontal range, $R = \frac{(u^2 \sin^2 \theta)}{g}$

$= \frac{40^2 \times \sin 2 \times 33.60}{9.8}$

$= \frac{1600 \times \sin 67.2)}{9.8}$

$= \frac{1600 \times 0.922}{9.8}$

$= 150.53\mathrm{m}.$

104. A cricketer can throw a ball to a maximum horizontal distance of 100m. How much high above the ground can the cricketer throw the same ball?

Ans. : Maximum horizontal distance, R = 100m

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is

$45^\circ, \mathrm{i.e.}, \theta = 45^\circ.$

The horizontal range for a projection velocity v, is given by the relation:

$\mathrm{R} = \mathrm{u}^2 \sin 2\theta / \mathrm{g}$

$100 = \mathrm{u}^2 \sin 90^\circ / \mathrm{g}$

$\frac{\mathrm{u}^2}{\mathrm{g}} = 100 \ldots (\mathrm{i})$

The ball will achieve the maximum height when it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration, a = -g

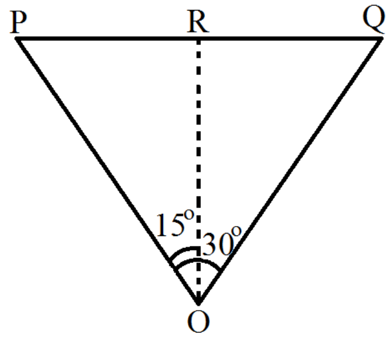
Using the third equation of motion:

$\mathrm{v}^2 - \mathrm{u}^2 = -2\mathrm{gH}$

$\mathrm{H} = \frac{\mathrm{u}^2}{2\mathrm{g}} = \frac{100}{2} = 50\mathrm{m}$

105. An aircraft is flying at a height of 3400m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0s apart is 30°, what is the speed of the aircraft?

Ans. : The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground, OR = 3400m

Angle subtended between the positions, $\angle \mathrm{POQ} = 30^\circ$

Time = 10s

In $\triangle \mathrm{PRO}$:

$\tan 15^\circ = \frac{\mathrm{PR}}{\mathrm{OR}}$

$\mathrm{PR} = \mathrm{OR} \tan 15^\circ$

$= 34000 \times \tan 15^\circ$

$\triangle \mathrm{PRO}$ is similar to $\triangle \mathrm{RQO}$.

$\therefore \mathrm{PR} = \mathrm{RQ}$

$\mathrm{PQ} = \mathrm{PR} + \mathrm{RQ}$

$= 2\mathrm{PR} = 2 \times 3400 \tan 15^\circ$

$= 6800 \times 0.268 = 1822.4\text{m}$

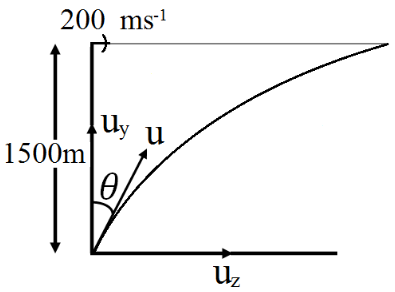
$\therefore$ Speed of the aircraft $= \frac{1822.4}{10} = 182.24\text{m/s}$

106. A fighter plane flying horizontally at an altitude of 1.5km with speed 720km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600ms^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10ms^{-2}).

Ans. : Height of the fighter plane = 1.5km = 1500m

Speed of the fighter plane, v = 720km/h = 200m/s

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, u = 600m/s

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

$\text{u}_\text{x}\text{t} = \text{vt}$

$\text{u} \sin\theta = \text{v}$

$\sin\theta = \frac{\text{v}}{\text{u}}$

$= \frac{200}{600} = \frac{1}{3} = 0.33$

$\theta = \sin^{-1}(0.33) = 19.5^\circ$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell.

$\therefore \text{H} = \frac{\text{u}^2 \sin^2(90-\theta)}{2\text{g}}$

$= \frac{(600)^2 \cos^2\theta}{2\text{g}}$

$= \frac{3,60,000 \times \cos^2 19.5}{2 \times 10}$

$= 16,006.482\text{m}$

$= 16\text{km}$

107. A particle starts from the origin at t = 0s with a velocity of $10.0\hat{\text{j}}\,\text{m/s}$ and moves in the x - y plane with a constant acceleration of $(8.0\hat{\text{i}} + 2.0\hat{\text{j}})\text{ms}^{-2}$. (a) At what time is the x-coordinate of the particle 16m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time?

Ans. : Velocity of the particle, $\vec{\nu} = 10.0\hat{\text{j}}\,\text{m/s}$

Acceleration of the particle, $\vec{a} = (8.0\hat{i} + 2.0\hat{j})$

Also,

But, $\vec{a} = \frac{d\vec{\nu}}{dt} = 8.0\hat{i} + 2.0\vec{j}$

$d\vec{\nu} = (8.0\hat{i} + 2.0\hat{j})dt$

Integrating both sides:

$\vec{\nu}(t) = 8.0t\hat{i} + 2.0t\hat{j} + \vec{u}$

Where,

$\vec{u}$ = Velocity vector of the particle at t = 0

$\vec{\nu}$ = Velocity vector of the particle at time t

Integrating the equations with the conditions:

at t = 0, r = 0, and at t = t, r = r

$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\hat{i} + \frac{1}{2} \times 2.0t^2\hat{j}$

$= \vec{u}t + 4.0t^2\hat{i} + t^2\hat{j}$

$= (10.0\hat{j})t + 4.0t^2\hat{i} + t^2\hat{j}$

$x\hat{i} + y\hat{j} = 4.0t^2\hat{i} + (10t + t^2)\hat{j}$

We observe that the motion of the particle is in x - y plane, So, on equating the coefficients of $\hat{i}$ and $\hat{j}$ we get,

$x = 4t^2$

$t = \left(\frac{x}{4}\right)^{1/2}$

and $y = 10t + t^2$

a. When y = 16m:

$t = \left(\frac{16}{4}\right)^{1/2} = 2s$

$\therefore y = 10 \times 2 + (2)^2 = 24m$

b. Velocity of the particle is given by:

$\vec{\nu}(t) = 8.0t\hat{i} + 2.0t\hat{j} + \vec{u}$

$\text{At } t = 2s$

$\vec{\nu}(2) = 8.0 \times 2\hat{i} + 2.0 \times 2\hat{j} + 10\hat{j}$

$= 16\hat{i} = 14\hat{j}$

$\therefore$ Speed of the particle,

$V = \sqrt{(16)^2 + (14)^2}$

$= \sqrt{256 + 196} = \sqrt{452}$

$= 21.26m/s$

108. The position of a particle is given by $r = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}\ m$ Where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find the v and a of the particle? (b) What is the magnitude and direction of velocity of the particle at t = 2.0s?

Ans. :

a. $\vec{\nu}(t) = (3.0\hat{i} - 4.0t\hat{J}); a = -4.0\hat{j}$

The position of the particle is given by:

$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\hat{J} + 4.0\hat{k}$

Velocity $\vec{\nu}$, of the particle given as:

$\vec{\nu} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\hat{J} + 4.0\hat{k})$

$\therefore \vec{\nu} = 3.0\hat{i} - 4.0t\hat{J}$

Acceleration, $\vec{a}$, of the particle is given as:

$\vec{a} = \frac{d\vec{\nu}}{dt} = \frac{d}{dt}(3.0\hat{i} - 4.0t\hat{J})$

$\therefore \vec{a} = -4.0\hat{J}$

b. 8.54m/s, 69.45° below the x - axis

We have velocity vector, $\vec{\nu} = 3.0\hat{i} - 4.0t\hat{J}$

$\text{At } t = 2.0s:$

$\vec{\nu} = 3.0\hat{i} - 8.0\hat{J}$

The magnitude of veocity is given by:

$|\vec{\nu}|\sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54m/s$

Direction, $\theta = \tan^{-1}\left(\frac{\nu_y}{\nu_x}\right)$

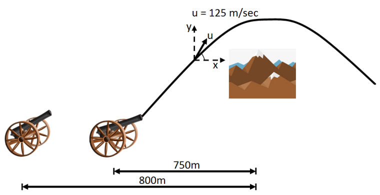
$= \tan^{-1}\left(\frac{-8}{3}\right) = -\tan^{-1}(2.667)$

$= -69.45^\circ$

The negative sing indicates that the direction of velocity is below the x - axis.

109. A hill is 500m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125m/ s over the hill. The canon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2m/ s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take g =10m/ s^2.

Ans. :



Speed of packets = 125m/ s

Height of hill = 500m

To cross the hill by packet the vertical components of the speed of packet (125m s^{-1}) must be minimised so that it can attain a height of 500m and the distance between Hill and Cannon must be half the range of packet.

$v^2 = u^2 + 2gh$

$0 = u_y^2 - 2gh$

$$u_y = \sqrt{2gh} = \sqrt{2 \times 10 \times 500}$$
$$= \sqrt{10000}$$
$$u_y = 100\mathrm{m/s}$$
$$u^2 = u_x^2 + u_y^2$$
$$(125)^2 = u_x^2 + 100^2$$
$$\Rightarrow \ u_x^2 = 125^2 - 100^2$$
$$u_x^2 = (125 - 100)(125 + 100)$$
$$= 25 \times 225$$
$$u_x = 5 \times 15$$
$$\Rightarrow \ u_x = 75\mathrm{m/s}$$

Vertical motion of packet

$$v_y = u_y + gt$$
$$0 = 100 - 10t$$

$\therefore$ Total time of $\frac{1}{2}$ flight $= 10\mathrm{sec}$ [Total time to reach the top of hill]

So the cannon must be at $\frac{1}{2}$ the range = horizontal distance in 10sec

$$= u_x \times 10 = 75 \times 10\mathrm{m} = 750$$

Hence, the distance between hill and cannon = 750m

So the distance to which cannon must move toward the hill = 800 - 750 = 50m

Time taken to move cannon in 50m $= \frac{\text{distance}}{\text{speed}} = \frac{50}{2} = 25\mathrm{sec}$

Hence, the total time taken by packet from 800m away from hill to reach other side

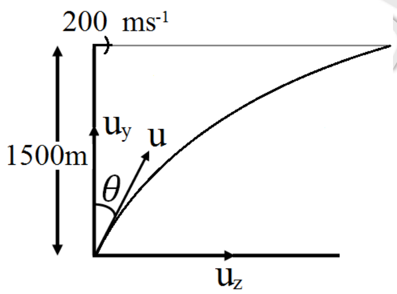
= 25s + 10s + 10s = 45 seconds.

110. A fighter plane flying horizontally at an altitude of 1.5km with speed 720km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed $600\mathrm{ms^{-1}}$ to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take $g = 10\mathrm{ms^{-2}}$).

Ans. : Height of the fighter plane = 1.5km = 1500m

Speed of the fighter plane, v = 720km/h = 200m/s

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, u = 600m/s

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

$u_x t = vt$

$u\sin\theta = v$

$\sin\theta = \frac{v}{u}$

$= \frac{200}{600} = \frac{1}{3} = 0.33$

$\theta = \sin^{-1}(0.33) = 19.5^\circ$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell.

$\therefore H = \frac{u^2\sin^2(90-\theta)}{2g}$

$= \frac{(600)^2\cos^2\theta}{2g}$

$= \frac{3{,}60{,}000\times\cos^2 19.5}{2\times10}$

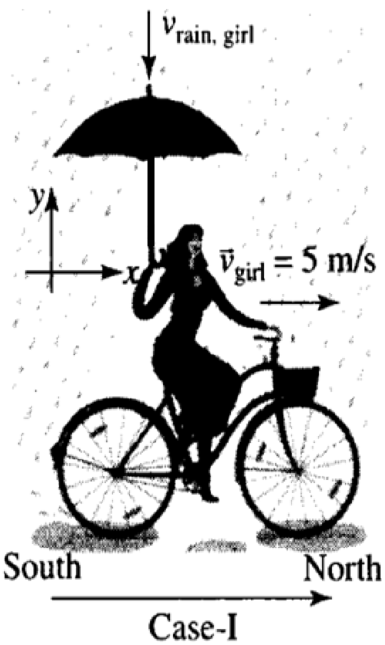
$= 16{,}006.482\mathrm{m}$

$= 16\mathrm{km}$

111. A girl riding a bicycle with a speed of 5m/ s towards north direction, observes rain falling vertically down. If she increases her speed to 10m/ s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer? (**Hint:** Assume north to be $\hat{i}$ direction and vertically downward to be $-\hat{j}$. Let the rain velocity v_r be $a\hat{i} + b\hat{j}$. The velocity of rain as observed by the girl is always V_r-V_{girl}. Draw the vector diagram/s for the information given and find a and b. You may draw all vectors in the reference frame of ground based observer)

Ans. : V_{rg} is the velocity of rain appears to the girl.

We must draw all vectors in the reference frame of ground-based observer.



Assume north to be $\hat{i}$ direction and vertically downward to be$(-\hat{j})$.

Let the rain velocity

$v_r = a\hat{i} + b\hat{j}$

Case I: According to the problem, velocity of

$girl = v_g = (5m/\ s)\hat{i}$

Let v_{rg} = velocity of rain w.r.t girl

$= v_r - v_g = (a\hat{i} + b\hat{j}) - 5\hat{i} = (a-5)\hat{i} + b\hat{j}$

According to question, rain appears to fall vertically downward.

Hence,$a - 5 = 0 \Rightarrow a = 5$

Case II: Now velocity of the girl after increasing her speed,

$v_g = (10m/\ s)\hat{i}$

$\therefore\ v_{rg} = v_r - v_g$

$= (a\hat{i} + b\hat{j}) - 10\hat{i} = (a-10)\hat{i} + b\hat{j}$

According to questions rain appears to fall at 45° to the vertical, hence

$\tan 45^\circ = \frac{b}{a-10} = 1$

$\Rightarrow \quad b = a - 10 = 5 - 10 = -5$

Hence, velocity of rain $= a\hat{i} + b\hat{j}$

$\Rightarrow \quad v_r = 5\hat{i} - 5\hat{j}$

Speed of rain

$= |v_r| = \sqrt{(5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}m/s$

112. Define angular velocity and angular acceleration. The total speed V_1 of a projectile at its greatest height is $\sqrt{\frac{6}{7}}$ of its speed V_2 when it is at half its greatest height. Show that the angle of projection is 30°.

Ans. : Angular Velocity: Angular velocity of an object in circular motion is defined as the time rate of change of is angular displacement. It is denoted by ω and is measured in radians per second (rad.s^{-1}).

$\omega = \frac{\text{angular displacement}}{\text{Time}} = \frac{\theta}{t} = \frac{d\theta}{dt}$

Angular Acceleration: Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity. It is denoted by 'a' and measured in rad s^{-2}.

$\alpha = \frac{\text{angular velocity change}}{\text{time taken}} = \frac{d\omega}{dt}$

Numerical: Velocity at highest point $= u\cos\theta = V_1$ (given)

$h_{max} = \frac{u^2 \sin^2\theta}{2g}$

Vertical velocity at $\frac{h_{max}}{2} = V_{2y} = \sqrt{u^2 \sin^2\theta - 2g\frac{h_{max}}{2}}$

$V_{2y} = \sqrt{u^2 \sin^2\theta\left(1 - \frac{1}{2}\right)} = \frac{u\sin\theta}{\sqrt{2}}$

$V_{2x} = u\cos\theta$

$V_2 = \sqrt{V_{2x}^2 + V_{2y}^2}$

$= \sqrt{u^2\cos^2\theta + \frac{u^2\sin^2\theta}{2}}$

Given, $V_1 = \sqrt{\frac{6}{7}}V_2$

$\therefore \frac{u\cos\theta}{u\sqrt{\cos^2\theta + \frac{\sin^2\theta}{2}}} = \sqrt{\frac{6}{7}}$

Squaring both the sides

$\frac{\cos^2\theta}{\cos^2\theta + \frac{\sin^2\theta}{2}} = \frac{6}{7}$

$\frac{1}{1 + \frac{\tan^2\theta}{2}} = \frac{6}{7}$

$1 + \frac{\tan^2\theta}{2} = \frac{7}{6} \Rightarrow \frac{\tan^2\theta}{2} = \frac{7}{6} - 1 = \frac{1}{6}$

$\tan\theta = \sqrt{\frac{2}{6}} = \frac{1}{\sqrt{3}}$

$\therefore \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ.$

113. A man whirls a stone round his head on the end of a string 4m long. Can the string be in a horizontal plane? If the stone has a mass of 0.4kg, and the string will break if the tension in it exceed 8N, what is the smallest angle the string can make with the horizontal ? What is the speed of the stone? (take g = $10ms^{-2}$)

Ans. : At equilibrium,

$T\cos\theta = mg;$

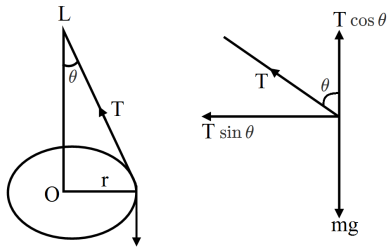
$T\sin\theta = \frac{mv^2}{r} = \frac{mv^2}{l\sin\theta}\ [\because r = l\sin\theta]$

As, $T\cos\theta = mg$; or $T = \frac{mg}{\cos\theta}$

In case the string becomes horizontal, $\theta = 90^\circ$,

$\therefore T = \frac{mg}{\cos 90^\circ} = \infty$

Thus, for making the string horizontal, the tension must be infinite which is impossible. Therefore, the string cannot be in horizontal plane.



The value of maximum angle θ is given by the breaking tension of the string, such that,

$$\mathrm{T}\cos\theta = \mathrm{mg};\ \mathrm{T}_{(\max)} = 8\mathrm{N}$$

$$\therefore\ \cos\theta = \frac{\mathrm{mg}}{\mathrm{T}_{(\max)}} = \frac{0.4\times10}{8} = \frac{1}{2}$$

or, $\theta = 60^\circ$

$\therefore$ Angle with the horizontal,

$$= 90^\circ - 60^\circ = 30^\circ$$

Also, $\mathrm{T}\sin\theta = \frac{\mathrm{mv}^2}{\mathrm{l}\sin\theta}$

or, $8\times\sin60^\circ = \frac{0.4\times\mathrm{v}^2}{4\times\sin60^\circ}$

or, $\mathrm{v} = \sqrt{80}\sin60^\circ \cong 7.7\mathrm{ms}^{-1}$

114. A cricket ball is thrown at a speed of 28m/ s in a direction 30° above the horizontal. Calculate:

a. The maximum height.
b. The time taken by the ball to return to the same level.
c. The horizontal distance from the point of projection to the point where the ball returns to the same level.

Ans. : Here, $\mathrm{u} = 28\ \mathrm{m/s}, \theta = 30^\circ$

a. **Maximum height:**

$$\mathrm{h} = \frac{\mathrm{u}^2\sin^2\theta}{2\mathrm{g}} = \frac{(28)^2(\sin30^\circ)^2}{2\times9.8}$$

$$= 10.0\mathrm{m}$$

b. **Time of flight:**

$$\mathrm{T} = \frac{2\mathrm{u}\sin\theta}{\mathrm{g}} = \frac{2\times28\times\sin30^\circ}{9.8}$$

$$= 2.9\ \mathrm{sec}$$

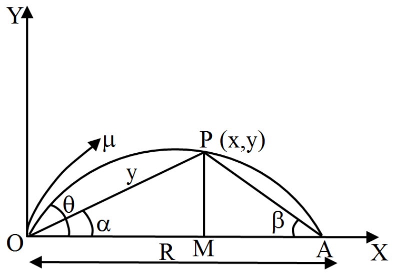
c. **Horizontal range:**

$$\mathrm{R} = \frac{\mathrm{u}^2\sin2\theta}{\mathrm{g}}$$

$$= (28)^2\times\frac{\sin(2\times30^\circ)}{9.8} = 69.28\mathrm{m}$$

115. A particle is thrown over a triangle from one end of a horizontal base that grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection; prove that: $\tan\theta = \tan\alpha + \tan\beta$.

Ans. : The statement in the question is shown in the diagram,



$\tan\alpha = \frac{y}{x}$ and $\tan\beta = \frac{y}{MA} = \frac{y}{R-x}$, where R is horizontal range.

$\therefore \ \tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$

$= \frac{(R-x+x)y}{x(R-x)} = \frac{yR}{x(R-x)}$

$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)} \ldots (i)$

Again, $x = (u\cos\theta)t \ldots (ii)$

$y = (u\sin\theta)t - \frac{1}{2}gt^2 \ldots (iii)$

From (ii) and (iii), we have

$y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos^2\theta\tan\theta}\right]$

Putting, $R = \frac{2u^2\sin\theta\cos\theta}{g}$

we get $y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos\theta\sin\theta}\right]$

$= x\tan\theta\left[1 - \frac{x}{R}\right]$

$\frac{y}{x} = \tan\theta\left(\frac{R-x}{R}\right) \ldots (iv)$

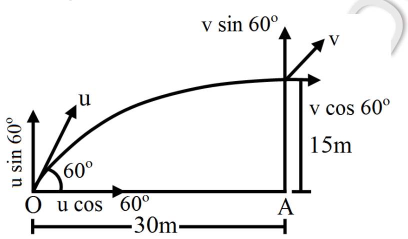
Putting (iv) in (i), we get

$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)} = \tan\theta$

$\therefore \ \tan\alpha + \tan\beta = \tan\theta$

116. A projectile shot at an angle of 60° above the horizontal ground strikes a vertical wall 30m away at a point 15m above the ground. Find the speed with which the projectile was launched and the speed with which it strikes the wall.

Ans. : Let the projectile be shot at angle 60° with velocity u. Velocity 'u' will have two components: Horizontal component.



$u_x = u\cos 60^\circ = \frac{30}{t}$

$t = \frac{60}{u}$

Vertical component

$u_y = u\sin 60^\circ$

Distance travelled by the projectile

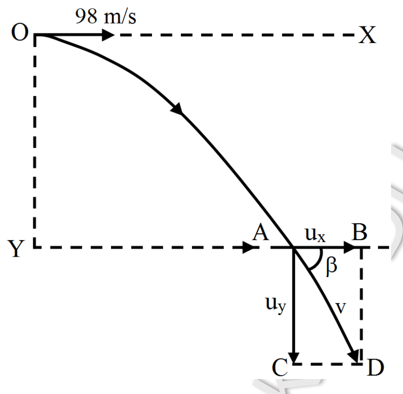
$$S = u_y t + \frac{1}{2}gt^2$$
$$15 = u\sin 60^\circ t - \frac{1}{2} \times 10t^2$$
$$15 = \frac{\sqrt{3}}{2}ut - 5t^2$$
$$15 = 30\sqrt{3} - 5\left(\frac{60}{u}\right)^2$$
$$u = \frac{18000}{(30\sqrt{3}-15)} = 22.07 \text{ m/s}$$
$\therefore$ Initial velocity of projectile = 22.07m/ s

Let the projectile strike the wall at point B above the ground with velocity 'V' which will have horizontal and vertical components.

Horizontal component
$$v_H = u\cos 60^\circ$$
$$= 22.07 \times \frac{1}{2} = 11.03 \text{ m/s}$$
Vertical component
$$v_V = u - gt$$
$$= 22.07 - 10 \times 2.72 = -5.13 \text{ m/s}$$
$$\left[\because\ t = \frac{60}{22.07} = 2.72 \text{ sec}\right]$$
Resultant velocity
$$v = \sqrt{v_V^2 + v_H^2}$$
$$= \sqrt{(-5.13)^2 + (11.03)^2}$$
$$v = 12.16 \text{ m/s}$$

117. A projectile is fired horizontally with a velocity of $98ms^{-1}$ from the hill 490m high. Find (i) time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the body strikes the ground.

Ans. :



Let OX and OY be two perpendicular axes and YO = 490m. A body projected horizontally from O with velocity u (= $98ms^{-1}$) meets the ground at A following a parabolic path shown in figure.

i. Let T be the time of flight of the projectile i.e. time taken by projectile to go from O to A.

Taking vertical downward motion (i.e. motion along OY axis) of projectile fiom O to A, we have

$y_0 = 0$, $y = 490m$, $u_y = 0$, $a_y = 9.8m/s^2$, $t = T$

As $y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$\therefore\ 490 = 0 + 0 \times T + \frac{1}{2} \times 9.8 \times T^2 = 4.9T^2$

$T = \sqrt{\frac{490}{4.9}} = 10s$

ii. Taking horizontal motion (i.e. motion along OX axis) of projectile from O to A, we have

$x_0 = 0$, $x = R$ (say), $u_x = 98m/s$, $t = T = 10s$, $a_x = 0$

As, $x = x_0 + u_x t + \frac{1}{2}a_x t^2$

$\therefore\ R = 0 + 98 \times 10 + \frac{1}{2} \times 0 \times 10^2 = 980m$

iii. Let v_x, v_y be the horizontal and vertical component velocity of the projectile at A.

Using the relation,

$v_x = u_x + a_x t = 98 + 0 \times 10 = 98m/s$

Represented by AB.

Using the relation,

$v_y = u_y + a_y t = 0 + 9.8 \times 10 = 98m/s$

Represented by AC.

$\therefore$ Resultant velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2}$

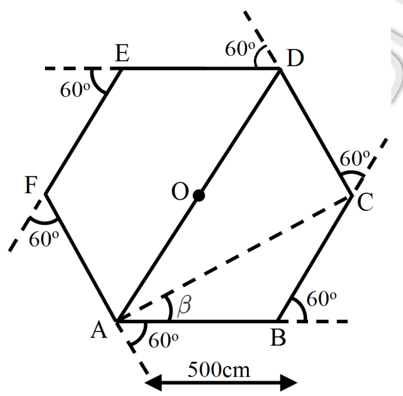
$= 98\sqrt{2}m/s$

If β is the angle which v makes with the horizontal direction then

$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1$ or $\beta = 45^\circ$ with the horizontal.

118. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Ans. : The distance after which motorist take a turn = 500m As motorist takes a turn at an angle of 60° each time, therefore motorist is moving on a regular hexagonal path. Let the motorist starts from point A and reaches at point D at the end of third turn and at initial point A at the end of sixth turn and at point C at the end of eighth turn.



Displacement of the motorist at the third turn = AD

= A + OD = 500 + 500 = 1000m

Total path length = AB + BC + CD

= 500 + 500 + 500 = 1500m

$$\therefore \frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{1000}{1500} = \frac{2}{3} = 0.67$$

At the sixth turn motorist is at the starting point A.

$\therefore$ Displacement of the motorist at the sixth turn = 0

Total path length = AB + BC + CD + DE + EF + FA

= 500 + 500 + 500 + 500 + 500 + 500 = 3000m

$$\therefore \frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{0}{3000} = 0$$

At the eighth turn, the motorist is at point C.

$\therefore$ Displacement of the motorist = AC

Using triangle law of vector addition,

$$\mathrm{AC} = \sqrt{\mathrm{AB}^2 + \mathrm{BC}^2 + 2\mathrm{AB.BC}\cos 60^\circ}$$

$$= \sqrt{(500)^2 + (500)^2 + 2 \times 500 \times 500 \times \tfrac{1}{2}}$$

$$= \sqrt{3 \times (500)^2} = 500\sqrt{3}\mathrm{m}$$

$$\mathrm{AC} = 500 \times 1.732\mathrm{m} = 866\mathrm{m}$$

If it is inclined at an angle β from the direction of AB,

then $\tan\beta = \frac{500 \sin 60^\circ}{500+500 \cos 60^\circ}$

$$= \frac{500\times\frac{\sqrt{3}}{2}}{500+500\times\frac{1}{2}} = \frac{500\times\frac{\sqrt{3}}{2}}{500\left(1+\frac{1}{2}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

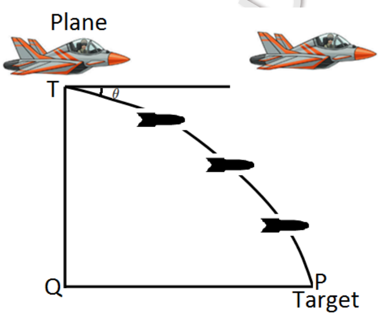
$$\beta = 30^\circ$$

$\therefore$ Displacement of the motorist at the end of eighth turn is 866m making an angle 30 with the initial direction of motion.

Total path length = 8 × 500 = 4000m

$$\therefore \frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8} = 0.22$$

119. A fighter plane is flying horizontally at an altitude of 1.5km with speed 720km/ h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?



Ans. :

Let pilot drops the bomb t sec before the point Q vertically up the target T.

The horizontal velocity of the bomb will to equal to the velocity of the fighter plane, but vertical component of it is zero. So in time t bomb must cover the vertical distance TQ as free fall with initial velocity zero.

$$u = 0, H = 1.5km = 1500m, g = +10m/s^2$$

$$H = ut + \frac{1}{2}gt^2$$

$$1500 = 0 + \frac{1}{2}10t^2$$

$$t = \sqrt{\frac{1500}{5}} = \sqrt{300} = 10\sqrt{3} \text{ second.}$$

$\therefore$ Distance covered by plane or bomb PQ = ut

$$PQ = 200 \times 10\sqrt{3} = 2000\sqrt{3}m$$

$$\tan\theta = \frac{TQ}{PQ} = \frac{1500}{2000\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{20\times3} = \frac{\sqrt{3}}{4}$$

$$\tan\theta = \frac{1.732}{4} = 0.433 = \tan^{-1} 23^\circ 42'$$

$$\Rightarrow \ \theta = 23^\circ 42'.$$

120. A ball is thrown vertically upwards with a velocity of 20m/ s from the top of a building of height 25m from the ground,

a. How high will the ball reach?

b. How long will it take for the ball to reach the ground?

c. Trace the trajectory of motion of this ball.

Ans. :

a. $h_1 = \frac{u^2}{2g} = \frac{20^2}{2\times10} = 20m$

Total height $= 25 + 20 = 45m$

b. $h = ut - \frac{1}{2}gt^2$

$$-25 = 20t - \frac{1}{2} \times 10 \times t^2$$

$$5t^2 - 20t - 25 = 0$$

$$(t-5)(t+1) = 0$$

$$\Rightarrow \ t = 5 \text{ sec.}$$

c. Trajectory will be a vertical line

$$y = ut + \frac{1}{2}gt^2$$

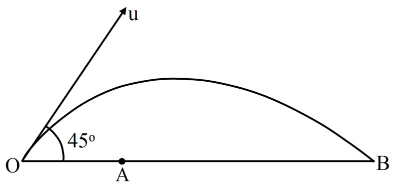
$$x = 0, y \neq 0$$

121. A gun kept on a straight horizontal road is used to hit a car travelling along the same road away from the gun with a uniform speed of 72km/ h. The car is at a distance of 500m from the gun when the gun is fired at an angle of 45° to the horizontal. Find,

i. The distance of the car from the gun when the shell hits it.

ii. The speed of projection of the shell from the gun.

Ans. : The gun and the car are at O and A respectively at t = 0 (fig below). Let us say that at t = t, the shell and the car reach B simultaneously so that the shell hits the car when it is at a distance OB from the gun.



Let u be the speed of projection of the shell from the gun. Then the initial horizontal component of the velocity of the shell $= \mathrm{u}\cos 45^\circ = \frac{\mathrm{u}}{\sqrt{2}}$ and the initial vertical component of the velocity of the shell $= \mathrm{u}\sin 45^\circ = \frac{\mathrm{u}}{\sqrt{2}}$.

Time of flight of the shell $= \frac{2\left(\frac{\mathrm{u}}{\sqrt{2}}\right)}{\mathrm{g}} = \sqrt{2}\left(\frac{\mathrm{u}}{\mathrm{g}}\right)$.

The car takes this time to cover the distance AB while the shell covers the distance OB in this time. But

OB = OA + AB = 500 + AB

Also, $\mathrm{OB} = \frac{\mathrm{u}}{\sqrt{2}}.\frac{\sqrt{2}\mathrm{u}}{\mathrm{g}} = \frac{\mathrm{u}^2}{\mathrm{g}}$

and $\mathrm{AB} = 20 \times \sqrt{2}\left(\frac{\mathrm{u}}{\mathrm{g}}\right) = 20\sqrt{2}\frac{\mathrm{u}}{\mathrm{g}}\ (\because 72\mathrm{km/\ h} = 20\mathrm{ms}^{-1})$

$\therefore\ \frac{\mathrm{u}^2}{\mathrm{g}} = 500 + 20\sqrt{2}\frac{\mathrm{u}}{\mathrm{g}}$

$\text{or}\ \ \mathrm{u}^2 - 20\sqrt{2}\mathrm{u} - 500 \times 9.8 = 0$

$\text{or}\ \ \mathrm{u}^2 - 20\sqrt{2}\mathrm{u} - 4900 = 0$

$\text{or}\ \ \mathrm{u} = \frac{20\sqrt{2}\pm\sqrt{400\times4+4\times4900}}{2}\mathrm{ms}^{-1}$

$= (10\sqrt{2} \pm \sqrt{5300})\mathrm{ms}^{-1}$

$= 10[\sqrt{2} \pm \sqrt{53}]\mathrm{ms}^{-1}$

$= 10[1.414 + 7.280]\mathrm{ms}^{-1} = 86.94\mathrm{ms}^{-1}$

This is the speed of projection of the shell from the gun. The distance of the car from the gun when the shell hits it is OB where,

$\mathrm{OB} = \frac{\mathrm{u}^2}{\mathrm{g}} = \frac{(86.94)^2}{9.8}\mathrm{m} \approx 771.3\mathrm{m}$

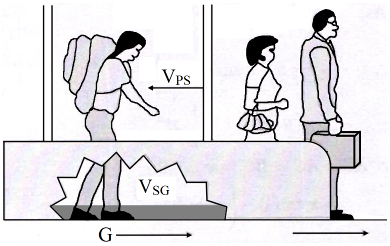
122. An airline passenger late for a flight walks on an airport moving sidewalk at a speed of 5.00km/ h relative to the sidewalk in the direction ofits motion. The sidewalkis moving at 3.00km/ h relative to the ground and has a total length of 135m.

i. What is the passenger's speed relative to the ground?
ii. How long does it take him to reach the end of the sidewalk?
iii. How much of the sidewalk has he covered by the time he reaches the end?

Ans. : The situation is sketched in figure. We assign a letter to each body in relative motion, P passenger, S sidewalk, G ground. The relative velocities V_{PS} and v_{SG} are given,

V_{PS} = 5.00km/ h, to the right

V_{SG} = 3.00km/ h, to the right



i. Here we must find the magnitude of the vector v_{PG}, given the magnitude and direction of two other vectors. We find the velocity v_{PG} by using the relation

$v_{PG} = v_{PS} + v_{SG}$

Here the vectors are parallel, and so the vector addition is quite simple see figure. We add vectors by adding magnitudes.

$v_{PG} = v_{PS} + v_{SG}$

= 5.00km/ h + 3.00km/ h

= 8.00km/ h

ii. The length of the sidewalk is 135m, and so this is the distance Δx_G the passenger travels relative to the ground. So, our problem is to find Δt when $\Delta x_G = 135m.$ The rate at which this distance along the ground is covered by the passenger is V_{PG}, where

$$v_{PG} = \frac{\Delta x_G}{\Delta t}$$

Therefore,

$$\Delta t = \frac{\Delta x_G}{v_{PG}} = \frac{135m}{8.00km/h\left(\frac{1.00m/s}{3.60km/h}\right)} = 60.8s$$

iii. The problem here is to determine how much of the sidewalk's surface the passenger moves over. If he was standing still and not walking along the surface, he would cover none of it. Because he is moving relative to the surface at velocity v_{PS}, he does move some distance Δx_s relative to the surface. The problem is to find Δx_s when $\Delta x = 60.8s,$ since we found in part (b) that this is the time interval during which he is on the moving sidewalk. His velocity relative to the sidewalk is $v_{PS} = \frac{\Delta x_s}{\Delta t}$, and so

$$\Delta x_s = v_{PS}\Delta t$$

$$= (5.00km/h)\left(\frac{1.00m/s}{3.60km/h}\right)(60.8s)$$

$$= 84.4m$$

123. A train starts from rest and moves with a constant acceleration of $2.0m/s^2$ for half a minute. The brakes are then applied and the train comes to rest in one minute. Find:

a. The total distance moved by the train.

b. The maximum speed attained by the train.

c. The position(s) of the train at half the maximum speed.

Ans. : Initial velocity u = 0

Acceleration $a = 2m/s^2$. Let final velocity be v (before applying breaks)

t = 30 sec

v = u + at ⇒ 0 + 2 × 30 = 60m/s

a. $S_1 = ut + \frac{1}{2}at^2 = 900m$

when breaks are applied u' = 60m/s

v' = 0, t = 60 sec (1 min)

Declaration $a' = \frac{(v-u)}{t} = \frac{0-16}{60} = -1m/s^2.$

$$S_2 = \frac{v'^2 - u'^2}{2a'} = 1800m$$

Total $S = S_1 + S_2$ = 1800 + 900 = 2700m = 2.7km.

b. The maximum speed attained by train v = 60m/s

c. Half the maximum speed $= \frac{60}{2} = 30m/s$

Distance $A = \frac{v^2 - u^2}{2a} = \frac{30^2 - 0^2}{2 \times 2} = 225m$ from starting point

When it accelerates the distance travelled is 900m. Then again declarates and attain 30m/s

$\therefore$ u = 60m/s, v = 30m/s, a = -1m/s^2

Distance $= \frac{v^2 - u^2}{2a} = \frac{30^2 - 60^2}{2(-1)} = 1350m$

Position is 900 + 1350 = 2250 = 2.25km from starting point.

124. A person is standing on a truck moving with a constant velocity of 14.7m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58.8m. Find the speed and the angle of projection:

a. As seen from the truck.

b. As seen from the road.

Ans. :

a. As seen from the truck the ball moves vertically upward comes back. Time taken = time taken by truck to cover 58.8m.

$\therefore \text{time} = \frac{s}{v} = \frac{58.8}{14.7} = 4\,\text{sec.}$ (V = 14.7m/s of truck)

u = ?, v = 0, g = -9.8m/s^2 (going upward), $t = \frac{4}{2} = 2\,\text{sec.}$

v = u + at

⇒ 0 = u - 9.8 × 2

⇒ u = 19.6m/s. (vertical upward velocity).

b. From road it seems to be projectile motion.

Total time of flight = 4 sec

In this time horizontal range covered 58.8m = x

$\therefore X = u\cos\theta t$

$\Rightarrow u\cos\theta = 14.7 \ldots (1)$

Taking vertical component of velocity into consideration.

$y = \frac{0^2 - (19.6)^2}{2 \times (-9.8)} = 19.6m$ [from(a)]

$\therefore y = u\sin\theta\, t - \frac{1}{2}gt^2$

$\Rightarrow 19.6 = u\sin\theta(2) - \frac{1}{2}(9.8)2^2$

$\Rightarrow 2u\sin\theta = 19.6 \times 2$

$\Rightarrow u\sin\theta = 19.6 \ldots (2)$

$\Rightarrow \frac{u\sin\theta}{u\cos\theta} = \tan\theta$

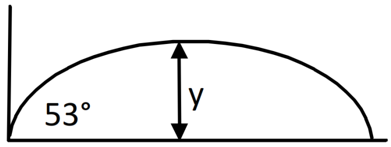
$\Rightarrow \frac{19.6}{14.7} = 1.333$

$\Rightarrow \theta = \tan^{-1}(1.333) = 53^\circ$

Again $u\cos\theta = 14.7$

$\Rightarrow u = \frac{14.7}{u\cos 53^\circ} = 24.42m/s.$

The speed of ball is 42.42m/s at an angle 53° with horizontal as seen from the road.



125. A ball is thrown horizontally from a point 100m above the ground with a speed of 20m/s. Find:

a. The time it takes to reach the ground.

b. The horizontal distance it travels before reaching the ground.

c. The velocity (direction and magnitude) with which it strikes the ground.

Ans. : It is a case of projectile fired horizontally from a height.

h = 100m, g = 9.8m/s^2

a. Time taken to reach the ground $\mathrm{t} = \sqrt{(2\mathrm{h}/\mathrm{g})}$

$= \sqrt{\frac{2\times100}{9.8}} = 4.51\mathrm{sec}.$

b. Horizontal range x = ut = 20 × 4.5 = 90m.

c. Horizontal velocity remains constant through out the motion.

At A, V_x = 20m/s

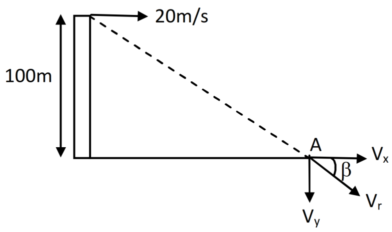
A V_y = u + at = 0 + 9.8 × 4.5 = 44.1m/s.

Resultant velocity $\mathrm{V_r} = \sqrt{(44.1)^2 + 20^2} = 48.42\mathrm{m/s}.$

$\tan\beta = \frac{\mathrm{V_y}}{\mathrm{V_x}} = \frac{44.1}{20} = 2.205$

$\Rightarrow \beta = \tan^{-1}(2.205) = 60^\circ.$

The ball strikes the ground with a velocity 48.42m/s at an angle 66° with horizontal.



126. It is 260km from Patna to Ranchi by air and 320km by road. An aeroplane takes 30 minutes to go from Patna to Ranchi whereas a delux bus takes 8 hours.

a. Find the average speed of the plane.

b. Find the average speed of the bus.

c. Find the average velocity of the plane.

d. Find the average velocity of the bus.

Ans. :

a. $\mathrm{V_{ave}}$ of plane$\left(\frac{\mathrm{Distance}}{\mathrm{Time}}\right)$

$= \frac{260}{0.5} = 520\mathrm{km/hr}.$

b. $\mathrm{V_{ave}}$ of bus $= \frac{320}{8} = 40\mathrm{km/hr}.$

c. plane goes in straight path

velocity $= \overrightarrow{\mathrm{V}}_{\mathrm{ave}} = \frac{260}{0.5} = 520\mathrm{km/hr}.$

d. Straight path distance between plane to Ranchi is equal to the displacement of bus.

$$\therefore \text{Velocity} = \overrightarrow{\text{V}}_{\text{ave}} = \frac{260}{8} = 32.5\text{km/hr.}$$

127. The benches of a gallery in a cricket stadium are 1m wide and 1m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110m from the batsman. On which bench will the ball hit?

Ans. : $\theta = 53^\circ$, so $\cos 53^\circ = \frac{3}{5}$

$\sec^2\theta = \frac{25}{9}$ and $\tan\theta = \frac{4}{3}$

Suppose the ball lands on nth bench

So, y = (n - 1)1 ...(1) [ball starting point 1m above ground]

Again $\text{y} = \text{x}\tan\theta - \frac{\text{gx}^2\sec^2\theta}{2\text{u}^2}$ [x = 110 + n - 1 = 110 + y]

$$\Rightarrow \text{y} = (110 + \text{y})\left(\frac{4}{3}\right) - \frac{10(110+\text{y})^2\left(\frac{25}{9}\right)}{2\times35^2}$$

$$\Rightarrow \frac{440}{3} + \frac{4}{3}\text{y} - \frac{250(110+\text{y})^2}{18\times35^2}$$

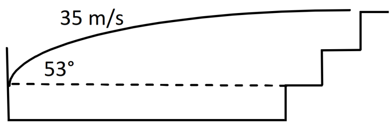
From the equation, y can be calculated.

$\therefore \text{y} = 5$

$\Rightarrow \text{n} - 1 = 5$

$\Rightarrow \text{n} = 6.$

The ball will drop in sixth bench.



128. A ball is dropped from a height of 5m onto a sandy floor and penetrates the sand up to 10cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform.

Ans. : Consider the motion of ball form A to B.

B → just above the sand (just to penetrate)

u = 0, a = 9.8m/s^2, s = 5m

$\text{S} = \text{ut} + \frac{1}{2}\text{at}^2$

$\Rightarrow 5 = 0 + \frac{1}{2}(9.8)\text{t}^2$

$\Rightarrow \text{t}^2 = \frac{5}{4.9} = 1.02$

$\Rightarrow \text{t} = 1.01$

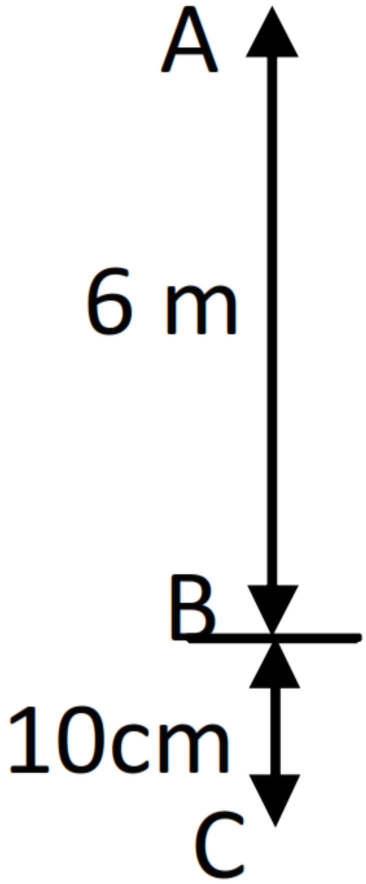
$\therefore$ velocity at B, v = u + at = 9.8 × 1.01 (u = 0) = 9.89m/s.

From motion of ball in sand

u_1 = 9.89m/s, v_1 = 0, a = ?, s = 10cm = 0.1m.

$$\text{a} = \frac{\text{v}_1^2 - \text{u}_1^2}{2\text{S}} = \frac{0-(9.89)^2}{2\times0.1} = -490\text{m/s}^2$$

The retardation in sand is 490m/s^2.



129. A car travelling at 60km/h overtakes another car travelling at 42km/h. Assuming each car to be 5.0m long, find the time taken during the overtake and the total road distance used for the overtake.

Ans. : v_1 = 60km/hr = 16.6m/s.

v_2 = 42km/h = 11.6m/s.

Relative velocity between the cars = (16.6 - 11.6) = 5m/s.

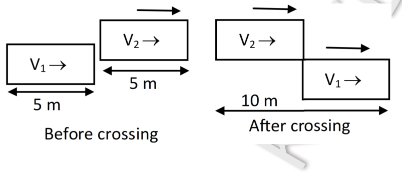
Distance to be travelled by first car is 5 + t = 10m.

Time $= \mathrm{t} = \frac{\mathrm{s}}{\mathrm{v}} = \frac{0}{5} = 2\,\mathrm{sec.}$ to cross the 2^{nd} car.

In 2 sec the 1^{st} car moved = 16.6 × 2 = 33.2m

H also covered its own length 5m.

∴ Total road distance used for the overtake = 33.2 + 5 = 38m.



130. A stone is thrown vertically upward with a speed of 28m/s.

a. Find the maximum height reached by the stone.
b. Find its velocity one second before it reaches the maximum height.
c. Does the answer of part.
d. Change if the initial speed is more than 28m/s suchas 40m/s or 80m/s?

Ans. : u = 28m/s, v = 0, a = -g = -9.8m/s^2

a. $\mathrm{S}=\frac{\mathrm{v}^2-\mathrm{u}^2}{2\mathrm{a}}=\frac{0^2-28^2}{2(9.8)}=40\mathrm{m}$

b. time $\mathrm{t}=\frac{\mathrm{v}-\mathrm{u}}{\mathrm{a}}=\frac{0-28}{-9.8}=2.85$

t' = 2.85 - 1 = 1.85

v' = u + at' = 28 - (9.8)(1.85) = 9.87m/s.

$\therefore$ The velocity is 9.87m/s.

c. No it will not change. As after one second velocity becomes zero for any initial velocity and deceleration is g = 9.8m/s^2 remains same. Fro initial velocity more than 28m/s max height increases.

* Case study based questions [16]

131. Read the passage given below and answer the following questions from i to v. we consider the motion of a projectile. An object that is in flight after being thrown or projected is called a projectile. Such a projectile might be a football, a cricket ball, a baseball or any other object. The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity. It was Galileo who first stated this independency of the horizontal and the vertical components of projectile motion in his Dialogue on the great world systems. **Horizontal range of a projectile:** The horizontal distance travelled by a projectile from its initial position (x = y = 0) to the position where it passes y = 0 during its fall is called the horizontal range, R. It is the distance travelled during the time of flight T_f . Therefore, the range R is $\mathrm{R}=(\mathrm{v}_o\cos\theta_o(\mathrm{T_f})$ $\mathrm{R}=\frac{(\mathrm{v_o}\cos\theta_\mathrm{o})(2\mathrm{v_o}\sin\theta_\mathrm{o})}{\mathrm{g}}$ $\mathrm{R}=\frac{(\mathrm{v_o^2}\sin\theta_\mathrm{o})}{g}$ This shows that for a given projection velocity, R is maximum when $2\theta_\mathrm{o}$ is maximum, i.e., when $\theta_\mathrm{o}=45^\circ$. The maximum horizontal range is, therefore $\mathrm{R}=\frac{\mathrm{v_o^2}}{g}$ **Maximum height of a projectile:** Maximum height that can be achieved during projectile and it is given by: $\mathrm{H_m}=\frac{(\mathrm{v_o}\sin\theta)^2}{2g}$

i. Range in projectile motion is maximum when θ° :
   a. 45^0
   b. 0^0
   c. 90^0
   d. None of these

ii. Who was first stated this independency of the horizontal and the vertical components of projectile motion in his Dialogue on the great world system?
   a. Galileo
   b. Newton
   c. Einstein
   d. None of these

iii. What is projectile motion?

iv. What is horizontal range of projectile? Give its formula:

v. What is maximum height of projectile? Give its formula:

Ans. :

i. (a) 45^0

ii. (a) Galileo

iii. The motion of object under only gravity force in the air is called projectile motion.

iv. The horizontal distance travelled by a projectile from its initial position to the position where it passes same horizontal position during its fall is called the horizontal range, R. It is the distance travelled during the time of flight T_f . Therefore, the range R is.

$$\mathrm{R \text{ is } R} = (\mathrm{v}_o \cos\theta_o(\mathrm{T_f})$$

$$\mathrm{R} = \frac{(\mathrm{v_o}\cos\theta_\mathrm{o})(2\mathrm{v_o}\sin\theta_\mathrm{o})}{\mathrm{g}}$$

$$\mathrm{R} = \frac{(\mathrm{v_o^2}\sin\theta_\mathrm{o})}{g}$$

v. Maximum height of a projectile: Maximum height that can be achieved during projectile and it is given by

$$\mathrm{H_m} = \frac{(\mathrm{v_o}\sin\theta)^2}{2g}$$

132. Read the passage given below and answer the following questions from 1 to 5. If A is vector given by A = Ax i + Ay j where The quantities A_x and A_y are called x, and y- components of the vector A. Note that Ax is itself not a vector, but A_x i is a vector, and so is A_y j. Using simple trigonometry, we can express A_x and A_y in terms of the magnitude of A and the angle θ it makes with the x-axis. $\mathrm{Ax} = \mathrm{A}\cos(\theta)$ $\mathrm{Ay} = \mathrm{A}\ \sin(\theta)$ If A and θ are given, Ax and Ay can be obtained using If Ax and Ay are given, A and θ can be obtained as follows – $\mathrm{A_x^2} + \mathrm{A_y^2} = (\mathrm{A}\cos\theta)^2 + (\mathrm{A}\sin\theta)^2$ $\mathrm{A_x^2} + \mathrm{A_y^2} = \mathrm{A^2}\cos^2\theta + \mathrm{A^2}\sin^2\theta \Rightarrow \mathrm{A_x^2} + \mathrm{A_y^2} = \mathrm{A^2}(\cos^2\theta + \sin^2\theta)$ $A_\mathrm{x}^2 + \mathrm{A_y^2} = \mathrm{A^2}(\because \sin^2\theta + \cos^2\theta = 1)\ \mathrm{A^2} = \mathrm{A_y^2} + \mathrm{A_y^2} \Rightarrow \mathrm{A} = \sqrt{\mathrm{A_x^2} + \mathrm{A_y^2}}\ldots$ $\text{Dividing } \mathrm{A_y} \text{ by } \mathrm{A_y}, \text{we get } \frac{\mathrm{A_y}}{\mathrm{A_x}} = \frac{\mathrm{A}\sin\theta}{\mathrm{A}\cos\theta} \Rightarrow \frac{\mathrm{A_y}}{\mathrm{A_x}} = \tan\theta\ \tan\theta = \frac{\mathrm{A_y}}{\mathrm{A_x}}$ $\theta = \tan^{-1}\left[\frac{\mathrm{A_y}}{\mathrm{A_x}}\right]$ **Position vector**-The position vector r of a particle P located in a plane with reference to the origin of an x-y reference frame is given by r = x i + y j where x and y are components of r along x-, and y- axes or simply they are the coordinates of the object. Suppose a particle moves along the Then, the displacement is: Δr = r_2-r_1. We can write this in a component form: Δr = (x' i + y' j) – (x i + y j) = iΔx – jΔy Where Δx = x' – x, Δy = y' – y. **The average velocity** (v) of an object is the ratio of the displacement and the corresponding time Interval. $\mathrm{V} = \frac{\triangle \mathrm{r}}{\triangle \mathrm{t}} = \frac{\mathrm{i}\triangle\mathrm{x} - \mathrm{j}\triangle\mathrm{y}}{\triangle\mathrm{t}}$ $= \mathrm{i} \times \frac{\triangle\mathrm{x}}{\triangle\mathrm{t}} + \mathrm{j} \times \frac{\triangle\mathrm{y}}{\triangle\mathrm{t}} = \mathrm{V_x i} + \mathrm{V_y j}$ So, if the expressions for the coordinates x and y are known as functions of time, we can use these equations to find vx and vy. The magnitude of v is then V = (v_x^2 + v_y^2) and the direction of v is given by the angle q and given by $\tan(\theta) = \frac{\mathrm{vx}}{\mathrm{vy}}$

i. If A is vector given by A = Ax i + Ay j .if the magnitude of vector is A and the angle θ it makes with the x-axis Ax can be given by:
   a. Ax = A cos(q)
   b. Ax = A sin(q)
   c. Ax = A tan(q)
   d. None of the above

ii. If A is vector given by A = Ax i + Ay j .if the magnitude of vector is A and the angle θ it makes with the x-axis Ay can be given by:

a. Ax = A cos(q)
b. Ax = A sin(q)
c. Ax = A tan(q)
d. None of the above

iii. Write a note on position vector and displacement of object:
iv. Write a note on average velocity:
v. If A is vector given by A = Ax i + Ay j where obtain expression for resultant amplitude of vector and its angle with x axis:

Ans. :

i. (a) Ax = A cos(q)
ii. (b) Ax = A sin(q)
iii. **Position vector-** The position vector r of a particle P located in a plane with reference to the origin of an x-y reference frame is given by r = x i + y j where x and y are components of r along x-, and y- axes. The displacement is: Δr = r_2-r_1. We can write this in a component form:

Δr = (x’ i + y’ j) – (x i + y j)

= iΔx – jΔy

Where Δx = x’ – x, Δy = y’– y.

iv. **The average velocity** (v) of an object is the ratio of the displacement and the corresponding time Interval.

$$\mathrm{V} = \frac{\triangle \mathrm{r}}{\triangle \mathrm{t}}$$
$$= \frac{\mathrm{i}\triangle \mathrm{x} - \mathrm{j}\triangle \mathrm{y}}{\triangle \mathrm{t}}$$
$$= \mathrm{i} \times \frac{\triangle \mathrm{x}}{\triangle \mathrm{t}} + \mathrm{j} \times \frac{\triangle \mathrm{y}}{\triangle \mathrm{t}}$$
$$= \mathrm{V_x i} + \mathrm{V_y j}$$

v. If A and θ are given, Ax and Ay can be obtained using If Ax and Ay are given, A and θ can be obtained as follows

$$\mathrm{A_x^2} + \mathrm{A_y^2} = (\mathrm{A}\cos\theta)^2 + (\mathrm{A}\sin\theta)^2$$
$$\mathrm{A_x^2} + \mathrm{A_y^2} = \mathrm{A}^2\cos^2\theta + \mathrm{A}^2\sin^2\theta$$
$$\Rightarrow \mathrm{A_x^2} + \mathrm{A_y^2} = \mathrm{A}^2(\cos^2\theta + \sin^2\theta)$$
$$A_{\mathrm{x}}^2 + \mathrm{A_y^2} = \mathrm{A}^2(\because \sin^2\theta + \cos^2\theta = 1)$$
$$\mathrm{A}^2 = \mathrm{A_y^2} + \mathrm{A_y^2}$$
$$\Rightarrow \mathrm{A} = \sqrt{\mathrm{A_x^2} + \mathrm{A_y^2}} \ldots$$
$$\text{Dividing } \mathrm{A_y} \text{ by } \mathrm{A_y}, \text{we get}$$
$$\frac{\mathrm{A_y}}{\mathrm{A_x}} = \frac{\mathrm{A}\sin\theta}{\mathrm{A}\cos\theta}$$
$$\Rightarrow \frac{\mathrm{A_y}}{\mathrm{A_x}} = \tan\theta$$
$$\tan\theta = \frac{\mathrm{A_y}}{\mathrm{A_x}}$$
$$\theta = \tan^{-1}\left[\frac{\mathrm{A_y}}{\mathrm{A_x}}\right]$$

133. A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72km/h. The jeep follows it at a speed of 90km/h, crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike?

Ans. : V_P = 90km/h = 25m/s.

V_C = 72km/h = 20m/s.

In 10 sec culprit reaches at point B from A.

Distance converted by culprit S = vt = 20 × 10 = 200m.

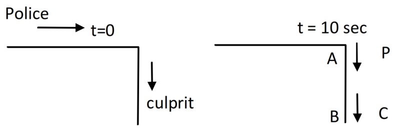
At time t = 10 sec the police jeep is 200m behind the culprit.

$\mathrm{Time} = \frac{\mathrm{s}}{\mathrm{v}} = \frac{200}{5} = 40\mathrm{s}.$ (Relative velocity is considered).

In 40s the police jeep will move from A to a distance S, where

S = vt = 25 × 40 = 1000m = 1.0km away.

∴ The jeep will catch up with the bike, 1km far from the turning.



134. A player hits a baseball at some angle. The ball goes high up in space. The player runs and catches the ball before it hits the ground. Which of the two (the player or the ball) has greater displacement?

Ans. : Same displacement.

----- “जिस दिन एक सिग्नेचर ऑटोग्राफ में बदलजाए तब मान लीजिएगा कि आप कामयाब हो गए ।” -----